Vibration Response of Tubular Joints Using a Simplified Mass-Spring Model

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***Abstract*—Using adhesives for joining tubular structures has been widely used to replace the traditional joining methods of welding, brazing, soldering, etc. The unique features associated with adhesives include low manufacturing cost, long components' life, and lightweight. The goal of this study is to investigate the vibration response of the tubular joints when they are subjected to a harmonic axial load considering that the shear stress is linear through the thickness. A simplified mass-spring model is applied to study the response of the problem analytically. Finite element method (FEM) using ANSYS is then availed to validate and compare results obtained in the analytical approach. Additionally, some parameters such as overlap length and adherent material will be changed to examine their influence on the frequency response. Results and findings achieved analytically and numerically showed that the natural frequencies increase as the adherent wave velocity increases, whereas they decrease as the overlap length increases.**

***Index Terms*—Adherent Properties, Analytical Solution, Natural Frequency, Tubular Joint.**

I. Introduction

There is often a complex deformation state for any structural material, such as tubular joints under concentrated linear shear stress, which may promote cracks or other defects to occur along the bonded joints. This problem is always accustomed to the non-metallic structures, especially in the situation when the traditional attachment techniques were used [1]. As a result, the use of adhesives in the joining of tubular structures has been steadily increasing [2, 3]. This use of adhesives comes amidst the urge to replace these old methods, such as brazing, welding, and soldering. The utilization of adhesives has been found to reduce the production cost, as well as to make components' life longer by decreasing the response to vibrations. The characteristics of these adhesive materials are remarkable, and they are regarded a limiting factor in bonding in many cases. As a result, these significant weaknesses led to the need to produce better adhesives and appropriate surface processing equipment to make adhesive surfaces effective [4, 5]. Despite these merits, the adhesive application is often used with precaution due to lack of reliable information regarding their behaviors when subjected to linear shear stress, especially that concerned Earlier study on tubular joints was done by Volkersen [6]. He has modeled with the assumption that adherent deformation takes place along the tensile axis, and adhesive is affected by only shear. The study by Kumar and Khan indicated that vibration-damping features increased when the flexible epoxy resin was used as adhesives [7]. Despite many merits, they are applied sparingly. This is because factors, such as porosity around voids response to dynamics and even adhesive thickness may impact its effectiveness [8]. Several pieces of research have been done to find out the impacts of displacement on the distribution of stress on the joints that are adhesively bonded together while considering viscoelastic, elastic, and viscoplastic materials. As a result, these studies revealed critical zones that may trigger failure in the joints.

In another important study, Abouel-Kasem, Hassab-Allah, and Nemat-Alla researched to find out the lifetime approximation of adhesively tubular joints that are bonded [9]. In the process, they adopted different types of geometries to conduct the investigation. They considered nonlinear viscoelastic adhesive characteristics. Then, the optimal geometry for the joints was established based on the lifetime and corresponding stresses for both the condition of open and closed ends [10]. The result indicated that high-stress value applied on an adhesive layer at the start of load application could indicate a dangerous scenario, specifically when the state of stress is followed by the cracks at the edge [9]. with long term impact, when subjected to various environmental conditions. In this paper, a simplified spring-mass model based on Lagrange's equation will be used to obtain the vibration response the tubular joint. This analytical solution will be then compared to a numerical solution using finite element method.

II. Methodology

Lagrange's equation for a simplified spring-mass model will be implemented to analytically study the vibration response of the adhesively bonded tubular joints subjected to a harmonic axial load. In this case, the tube is kept fixed at the left edge. A unit force is applied at the right edge of the tube, properties will change along the x-axis only (Figs. (1) and (2).

Each mass will write:

$$m_1 = \rho_1 \pi ({R_4}^2 - {R_3}^2) l_1 \tag{1}$$

$$m_2 = \rho_2 \pi ({R_4}^2 - {R_3}^2) l_o \tag{2}$$

$$m_3 = \rho_a \pi ({R_3}^2 - {R_2}^2) l_o \tag{3}$$

$$m_4 = \rho_2 \pi ({R_2}^2) l_o \tag{4}$$

$$m_5 = \rho_2 \pi ({R_2}^2) l_2 \tag{5}$$

Each spring constant will write:

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$$k_1 = \frac{E_1 \pi (R_4{}^2 - R_3{}^2)}{l_1} \tag{6}$$

$$k_2 = \frac{E_1 \pi (R_4{}^2 - R_3{}^2)}{l_o} \tag{7}$$

$$k_3 = \frac{G_a \rho_o 2\pi \left(\frac{R_3 + R_2}{2}\right)}{R_3 - R_2} \tag{8}$$

$$k_4 = \frac{E_2 \pi \left(R_2{}^2\right)}{l_o} \tag{9}$$

$$k_5 = \frac{E_2 \pi \left(R_2{}^2\right)}{l_2} \tag{10}$$

Kinetic energy equation:

$$K = \frac{1}{2} m_1 \dot{x}_1{}^2 + \frac{1}{2} m_2 \dot{x}_2{}^2 + \frac{1}{2} m_3 \dot{x}_3{}^2 + \frac{1}{2} m_4 \dot{x}_4{}^2 + \frac{1}{2} m_5 \dot{x}_5{}^2$$

Potential energy equation:

$$U = \frac{1}{2} k_1 x_1{}^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{1}{2} k_3 (x_3 - x_2)^2 + \frac{1}{2} k_4 (x_4 - x_3)^2 + \frac{1}{2} k_5 (x_5 - x_4)^2$$

Lagrange's equation can be expressed as:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial x_i}\right) + \frac{\partial U}{\partial x_i} = P \tag{11}$$

Differential equations that describe the equations of motion of the system is as follows:

$$x_1 : m_1 \ddot{x_1} + k_1 x_1 - k_2 (x_2 - x_1) = 0 \tag{12}$$

$$x_2 : m_2 \ddot{x_2} + k_2 (x_2 - x_1) - k_3 (x_3 - x_2) = 0 \tag{13}$$

$$x_3 : m_3 \ddot{x_3} + k_3 (x_3 - x_2) - k_4 (x_4 - x_3) = 0 \tag{14}$$

$$x_4 : m_4 \ddot{x_4} + k_4 (x_4 - x_3) - k_5 (x_5 - x_4) = 0 \tag{15}$$

$$x_5 : m_5 \ddot{x_5} + k_5 (x_5 - x_4) = P \tag{16}$$

A matrix in the form of $[m]\ddot{x} + [K]x = [P]$ will be structured in MATLAB to obtain the natural frequency response.

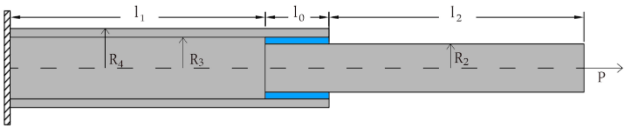

Fig.1. Schematic diagram of the tubular bonded joint

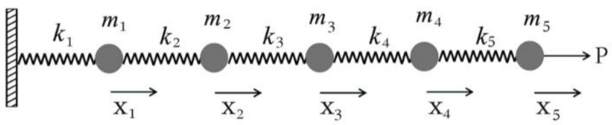

Fig.2. Spring-mass system for the tubular joint

Finite element method (FEM) will be implemented using ANSYS to investigate the response of the tubular joints when they are subjected to a harmonic axial load. The purpose of using the FEM is to validate and compare results obtained in the analytical approach. For simplicity purposes, the numerical approach will deal with the problem as a 2-D finite element model. Adherents and adhesive are isotropic. The adherents are assumed to be 6061-T6 aluminum with elastic modulus of 69 GPa with a density of 2710 kg/m3 while the adhesive is taken to be epoxy with shear modulus of 791 MPa and a density of 1200 kg/m3. The thickness of the outer and inner adherents are 17.8 mm and 6.4 mm, respectively whereas the thickness of adhesive is 1.2 mm. The lengths of the outer and inner adherents are 250 mm, and the overlap length is assumed to be 50 mm. Mesh element size is selected to be 0.6 mm. Inner adherent is assumed to be fixed support, and the motions of the entire geometry is limited to only take effect in the x-axis. A harmonic load is applied to the right side of the outer adherent. Other parameters such as overlap length and the type of adherent material are considered as well

III. Results and Discussion

The results will be based on three different scenarios. Firstly, the adherents are considered of aluminum with the mechanical properties mentioned in the previous section, and the overlap length is taken to be 50 mm. Secondly, different materials are considered for adherents with keeping the overlap unchanged, 50mm. Lastly, different overlap length will be applied to assess the effects of geometry on the natural frequency.

A. *Tubular joints of aluminum adherents*

The mass-spring model is in good agreement with the finite element model (Table 1). The natural frequency positions are well predicted where the first natural frequency is ~ 2.5 kHz (Fig. 3).

TABLE I: Natural Frequencies Obtained from Spring-Mass Model and ANSYS for Aluminum Adherents

Mode	Spring-mass model (Hz)	ANSYS (Hz)	Error %
1	2060	2552.5	23.91
2	5254	8031.5	52.86
3	13150	13120	0.23

TABLE II: Mechanical Properties of Adherent Materials

Material	E (Gpa)	G (Gpa)	ρ (kg/m^3)
Aluminum	69	26.5	2710
Steel	209	80.4	7550
CFRP	138	6.3	1800
GFRP	35	4	1600

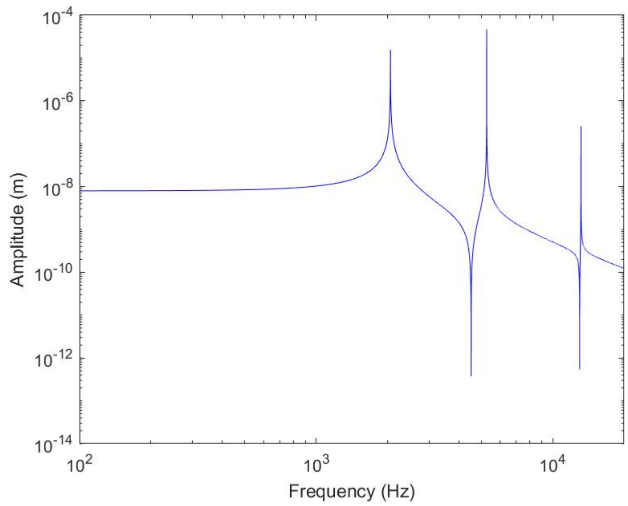

Fig.3. Natural frequencies of spring-mass system of aluminum adherent

B. Tubular joints of different adherents

The effects of adherent materials on the natural frequency is investigated. The spring-mass and ANSYS models are in good agreement for all the materials selected (Tables 2 and 3). It can be noted that the effects of adherent material show competition between stiffness and inertia. The highest natural frequencies are obtained with CFRP adherents. This ranking of adherents in terms of natural frequency is based on the value of the wave velocity of adherents $v_{o,i} = \sqrt{\frac{E_{o,i}}{\rho_{o,i}}}$ In other words, higher the value of wave velocity of adherents leads to higher natural frequencies. Aluminum, steel, and GFRP have comparable values of wave velocity and hence, have relatively close natural frequencies. Consequently, the value of natural frequency relies heavily on the wave velocity of adherent.

TABLE III: NATURAL FREQUENCIES OBTAINED FROM SPRING-MASS MODEL AND ANSYS FOR DIFFERENT ADHERENTS

Mode	Spring-mass model (Hz)	ANSYS (Hz)	Error %
	Aluminum		
1	2060	2552.5	23.91
2	5254	8031.5	52.86
3	13150	13120	0.23
	Steel		
1	2103	2570.2	22.22
2	5328	8069.5	51.45
3	13050	13098	0.37
	CFRP		
1	3531	4340.9	22.94
2	8974	13646	52.06
3	22081	22233	0.69
	GFRP		
1	1918	2385.6	24.38
2	4897	7509	53.34
3	12310	12277	0.27

TABLE IV: NATURAL FREQUENCIES OF ALUMINUM ADHERENTS FOR DIFFERENT OVERALP LENGTHS

Mode	Spring-mass model (Hz)	ANSYS (Hz)	Error %
	l_o = 25 mm		
1	2197	2678	21.89
2	5583	8418.7	50.79
3	20420	13665	33.08
	l_o = 50 mm		
1	2060	2552.5	23.91
2	5254	8031.5	52.86
3	13150	13120	0.23
	l_o = 75 mm		
1	1915	2404.6	25.57
2	4975	7590.5	52.57
3	10110	12542	24.06
	l_o = 100 mm		
1	1778	2268.9	27.61
2	4747	7205	51.78
3	8511	12056	41.65

C. Changing the overlap length

The effects of geometrical parameters on the natural frequency is investigated. Adherent material is taken to be aluminum while considering different overlap lengths of 25, 50, 75, and 100 mm. By comparing the mathematical results of Lagrange's equation to those obtained numerically in ANSYS, it can be noted that an increase in the overlap length leads to a decrease in the natural frequencies (Table 4). This is because the axial stiffness of the adherents is decreased due to the increase in the overlap length which leads to a reduction of the joint's stiffness.

IV. CONCLUSION

Using adhesives for joining tubular structures has been widely used to replace the traditional joining methods of welding, brazing, soldering, etc. The goal of this study was to investigate the vibration response of the tubular joints when they are subjected to a harmonic axial load considering that the shear stress is linear through the thickness. Analytical approach of Lagrange's equation for a simplified spring-mass system was developed to investigate the frequency response. Finite element method (FEM) using ANSYS was then availed to validate and compare results obtained in the analytical approach. Results achieved analytically and numerically were found to conform. A parametric study was conducted to investigate the influence of geometrical and material parameters on the frequency response. Therefore, the following conclusion can be drawn:

- The natural frequencies increase as the adherent wave velocity increases.
- The natural frequencies decrease as the overlap length increases.

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