An Optimal Economic Dispatch Algorithm for Large Scale Power Systems with Cogeneration Units

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Abstract—This paper proposes an optimal economic dispatch algorithm for large scale power systems with cogeneration units. A hybrid differential evolution with multiplier updating (HDE-MU) is introduced to solve the combined heat and power economic dispatch (CHPED) problems. The hybrid differential evolution (HDE) has the ability to efficiently search and actively explore solutions. Multiplier updating (MU) is introduced to avoid deforming the augmented Lagrange function (ALF), which is adopted to manage system constraints of the CHPED problem. The proposed HDE-MU integrates the HDE with the MU. A practical CHPED system with two cases of a basic case and a large case is employed to demonstrate that the proposed algorithm has the better performance in comparison with the previous methods, and the proposed HDE-MU has the effectiveness when applied to the large scale CHPED operation of power systems.

Index Terms—Cogeneration, differential evolution, economic dispatch, multiplier updating, large scale.

I. INTRODUCTION

The conversion of fossil fuel into electricity is not an efficient process. Even the energy efficiency of the most modern combined cycle plants are between 50 and 60% efficient [1]. Most of the energy wasted in the conversion process is heat. Recently, combined heat and power (CHP) units have played an increasingly important role in the utility industry [2~ 7]. Complexity arises if one or more units produce both power and heat. Such a case, both the power demand and the heat demand must be satisfied. Cogeneration units provide both electrical power and heat to customers. The heat production capacity of most cogeneration units, depend on the power generated, and vice versa. The mutual dependency of multiple-demand and Heat-Power capacity of those units introduce complexities in the integration of cogeneration units into the economic dispatch problem. Solving such a complex optimization problem requires powerful techniques. Non-linear optimal algorithms, such as dual and quadratic programming [8], and gradient descent approaches, such as Lagrangian relaxation [9], have been applied for solving CHPED problems. However, these methods cannot handle non-smooth non-convex fuel cost function of the conventional thermal generator. The advent of stochastic search algorithms has overcome this problem for solving CHPED problems. Group search optimization [1], cuckoo search algorithm [2], real coded genetic algorithm with improved

Mühlenbein mutation [3], crisscross optimization algorithm [4], oppositional teaching learning based optimization [5], hybrid harmony search with arithmetic crossover operation [6], improved group search optimization [7], Generalized algebraic modeling system (GAMS) [10], canonical coordinates method (CCM) [11], harmony search with genetic algorithm (HSGA) [12], harmony search for economic dispatch (EDHS) [13], cuckoo optimization algorithm with penalty function (PFCOA) [14], gravitational search algorithm (GSA) [15], exchange market algorithm (EMA) [16], benders decomposition (BD) [17], particle swarm optimization with time varying acceleration coefficients (TVAC-PSO) [18], firefly algorithm (FA) [19], invasive weed optimization (IWO) [20], and harmony search (HS) [21] have been applied to solve the CHPED problems. The results obtained by the proposed algorithm, involving HDE and MU, have been compared with the existing methodologies reported in recent literature.

Storn and Price [22] developed the Differential evolution (DE) which immediately gained popularity as a robust evolutionary algorithm. DE has been widely applied to the optimization problems in the power systems [23~ 27]. Throughout the years, DE has been used extensively for optimization problems, many results of which are the best compared to other standard methodologies. However, it has problems of converging onto local optimal solutions. A migration [28] is embedded in the proposed HDE to overcome such drawbacks for solving CHPED problems. The migration operation is included in the HDE to regenerate a newly diverse population, which prevents individuals from gradually clustering and thus greatly increases the amount of search space explored for a small population.

The reminder of this paper is organized as follows: Section II introduces the mathematical formulation of the CHPED problem. Section III describes the proposed algorithm involving HDE and MU. Section IV presents two cases of one basic case and one large scale case for testing the CHPED problem, compares the results obtained by HDE-MU with other methods reported in the literature. Conclusions are finally in Section V.

II. FORMULATION OF THE CHPED PROBLEM

A. Objective function

The CHPED problem is to determine the unit power and heat production so that the system's production cost is minimized while the power and heat demands and other constraints are met. It can be mathematically stated as,

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$$\text{Minimize} \quad \sum_{i=1}^{n_p} \text{Cost}_i(P_i) + \sum_{j=1}^{n_b} \text{Cost}_j(H_j, P_j) + \sum_{k=1}^{n_h} \text{Cost}_k(H_k) \quad (1)$$

Where

$$\text{Cost}_i(P_i) = a_i + b_i P_i + c_i P_i^2 + d_i P_i^3 \quad (2)$$

$$\text{Cost}_j(H_j, P_j) = a_j + b_j P_j + c_j P_j^2 + d_j H_j^2 + r_j H_j + w_j P_j H_j \quad (3)$$

$$\text{Cost}_k(H_k) = a_k + b_k H_k + c_k H_k^2 \quad (4)$$

and where P stands unit power generation; H is unit heat production; $\text{Cost}_i(P_i)$ with $i=1,2,\dots, n_p$ represents the cost function of the i^{th} power-only unit in \$/h and a_i, b_i, c_i , and d_i are cost coefficients of generator i ; $\text{Cost}_j(H_j, P_j)$ with $j=1,2,\dots, n_b$ is the fuel cost function of the j^{th} CHP unit in \$/h and a_j, b_j, c_j, d_j, r_j , and w_j are cost coefficients of unit j ; $\text{Cost}_k(H_k)$ with $k=1,2,\dots, n_h$ represents the cost function of the k^{th} heat-only unit in \$/h and a_k, b_k , and c_k are cost coefficients; n_p, n_b , and n_h denote the number of power-only units, co-generation units, and heat-only units, respectively; subscripts of i, j , and k are used for above mentioned unit; it should be mentioned that (P_i, P_j) and (H_j, H_k) are the output active power (in MW) and heat production (in MWth), respectively.

B. Constraints

Subject to the equilibrium constraints of electricity and heat production, and the capacity limits of each unit. Power and heat balance equations are represented by equality constraints (5) and (6), respectively.

$$\sum_{i=1}^{n_p} P_i + \sum_{j=1}^{n_b} P_j = P_d \quad (5)$$

$$\sum_{j=1}^{n_b} H_j + \sum_{k=1}^{n_h} H_k = H_d \quad (6)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad i = 1, \dots, n_p \quad (7)$$

$$P_j^{\min}(H_j) \leq P_j \leq P_j^{\max}(H_j), \quad j = 1, \dots, n_b \quad (8)$$

$$H_j^{\min}(P_j) \leq H_j \leq H_j^{\max}(P_j), \quad j = 1, \dots, n_b \quad (9)$$

$$H_k^{\min} \leq H_k \leq H_k^{\max}, \quad k = 1, \dots, n_h \quad (10)$$

Where P_d and H_d are system power and heat demands, respectively. $P^{\min}$ and $P^{\max}$ are unit power capacity limits; $H^{\min}$ and $H^{\max}$ are unit heat capacity limits.

The CHPED problem clearly introduces the complication of more constraints than in required pure power economic dispatch problem. The insufficiencies difficulties with conventional methods thus follow from the fact that CHPED is a highly constrained optimization problem.

III. THE PROPOSED HDE-MU

A. DE

The DE algorithm is one of the population-based optimization algorithms. The steps for implementing DE are as follows [22]:

Step 1: Initial population: A population of N_p initial solutions randomly distributed in the n_c dimensional search space of the optimization problem, are initiated. The DE uses N_p vectors of variables x in the optimization problem, namely, $x^G = \{x_i^G, i = 1, \dots, N_p\}$, as a population in generation G . For convenience, the decision vector, x_i , is represented as $(x_{i1} \dots x_{ji} \dots x_{n_{ci}})$. Here, the decision variable, x_{ji} is directly coded as a real value within its bounds of $(x_{ji}^{\min}, x_{ji}^{\max})$. Each individual is generated as follows:

$$x_{ji}^G \Big|_{G=0} = x_{ji}^{\min} + \text{rand}(0,1) * (x_{ji}^{\max} - x_{ji}^{\min}) \quad (11)$$

$$j = 1, 2, \dots, n_c, \quad i = 1, 2, \dots, N_p$$

Where $\text{rand}(0, 1)$ is a random number between 0 and 1.

Step 2: Mutation operator: In mutation step, for each individual x_i (target vector) of the new population, three different individuals x_{r1} , x_{r2} , and x_{r3} ($r1 \neq r2 \neq r3 \neq i$) are pseudo-randomly extracted from the population to generate a new vector as:

$$z_{ji} = x_{jr1} + F * (x_{jr2} - x_{jr3}) \quad j = 1, 2, \dots, n_c \quad (12)$$

Where $F \in [0,2]$ is a uniformly distributed random number which controls the length of the population exploration vector $(x_{r2} - x_{r3})$.

Step 3: Crossover operator: After mutation step, the crossover operator, according to the following equation, is applied on the mutation vector Z_{ji} and the vector x_{ji} to generate the trial vector U_{ji} , for increasing population diversity of the mutation vector.

$$U_{ji} = \begin{cases} z_{ji}, & \text{if } \text{rand}(0,1) \leq CR \\ x_{ji}, & \text{otherwise} \end{cases} \quad (13)$$

$$j = 1, 2, \dots, n_c, \quad i = 1, 2, \dots, N_p$$

Where $CR \in [0,1]$ is known as the crossover rate which is a constant.

Step 4: Selection & evaluation operator: The selection & evaluation process is repeated for each pair of target/trial vectors using the evaluation function $F(U_{ji})$ to compare with the evaluation function value $F(x_{ji})$, and the better one will be selected to be a member of the DE population generation for the next iteration (x_{ji}^{G+1}).

B. HDE

In HDE [23~ 27], the one-to-one competition will have a faster convergence speed to give a higher probability toward a global optimum with much less computation time. It can use a small population in the evolutionary process to obtain a global solution. Generally, Evolutionary optimization

involves two critical issues evolutionary direction and population diversity. As the evolutionary direction is effective in searching, the strong evolutionary direction can reduce the computational burden and increase the probability of rapidly finding an (possibly local) optimum. As population diversity is increased, the genotype of the offspring differs more from the parent. Accordingly, a highly diverse population can increase the probability of exploring the global optimum and prevent a premature convergence to a local optimum. These two important factors are here balanced by employing the migration [28] into HDE that can determine an efficacious direction in which to search for a solution and simultaneously maintain an appropriate diversity for a small population. The migrant individuals are generated based on the best individual, $x_b^{G+1} = (x_{1b}^{G+1}, x_{2b}^{G+1}, \dots, x_{n_{cb}}^{G+1})$, by non-uniformly random choice. Genes of the i^{th} individual are regenerated according to

$$x_{ji}^{G+1} = \begin{cases} x_{jb}^{G+1} + \rho(x_j^{\min} - x_{jb}^{G+1}), & \text{if } r_1 < \frac{x_{jb}^{G+1} - x_j^{\min}}{x_j^{\max} - x_j^{\min}} \\ x_{jb}^{G+1} + \rho(x_j^{\max} - x_{jb}^{G+1}), & \text{otherwise} \end{cases} \quad (14)$$

where $j = 1, \dots, n_c$; $i = 1, \dots, N_p$, r_1 and ρ are random numbers in the range of [0,1]. The migration may be performed if only the best fitness has not been improved for over 500 generations running, and the migrant population will not only become a set of newly promising solutions but also easily escape the local extreme value trap. More details of the HDE have shown in [23~ 27].

C. MU

Michalewicz et al. [29] surveyed and compared several constraint-handling techniques used in evolutionary algorithms. Among these techniques, the penalty function method is one of the most popularly used to handle constraints. In this method, the objective function includes a penalty function that is composed of the squared or absolute constraint violation terms. Powell [30] noted that classical optimization methods include a penalty function have certain weaknesses that become most serious when penalty parameters are large. More importantly, large penalty parameters ill condition the penalty function so that obtaining a good solution is difficult. However, if the penalty parameters are too small, the constraint violation does not contribute a high cost to the penalty function. Accordingly, choosing appropriate penalty parameters is not trivial. Herein, the MU [31] is introduced to handle this constrained optimization problem. Such a technique can overcome the ill conditioned property of the objective function. Considering the nonlinear problem with general constraints as follows: Where $h_k(x)$ and $g_k(x)$ stand for equality and inequality constraints, respectively.

$$\begin{aligned} & \min f(x) \\ \text{subject to } & h_k(x) = 0, \quad k = 1, \dots, m_e \\ & g_k(x) \leq 0, \quad k = 1, \dots, m_i \end{aligned} \quad (15)$$

Fig.1 The flow chart of the HDE-MU
*N_o : maximum number of iterations of outer loop
**N_i : maximum number of iterations of inner loop

The augmented Lagrange function (ALF) [30] for constrained optimization problems is defined as:

$$\begin{aligned} L_a(x, \nu, v) = & f(x) + \sum_{k=1}^{m_e} \alpha_k \{ [h_k(x) + v_k]^2 - v_k^2 \} \\ & + \sum_{k=1}^{m_i} \beta_k \{ \langle g_k(x) + v_k \rangle_+^2 - v_k^2 \} \end{aligned} \quad (16)$$

where α_k and β_k are the positive penalty parameters, and the corresponding Lagrange multipliers $\nu = (\nu_1, \dots, \nu_{m_e})$ and $v = (v_1, \dots, v_{m_i}) \geq 0$ are associated with equality and inequality constraints, respectively.

The contour of the ALF does not change shape between generations while constraints are linear. Therefore, the contour of the ALF is simply shifted or biased in relation to the original objective function, $f(x)$. Consequently, small penalty parameters can be used in the MU. However, the shape of contour of L_a is changed by penalty parameters while the constraints are nonlinear, demonstrating that large penalty parameters still create computational difficulties. Adaptive penalty parameters of the MU are employed to alleviate the above difficulties. More details of the MU have shown in [31].

D. The proposed HDE-MU

Figure 1 displays the flow chart of the proposed algorithm, which has two iterative loops. The ALF is used to obtain a minimum value in the inner loop with the given penalty parameters and multipliers, which are then updated in the outer loop toward producing an upper limit of L_a . When both inner and outer iterations become sufficiently large, the ALF converges to a saddle-point of the dual problem [29]. Advantages of the proposed HDE-MU are that the HDE efficiently searches the optimal solution in the economic dispatch process and the MU effectively tackles

system constraints.

IV. SYSTEM SIMULATIONS

In this section, the proposed method is tested on two cases of one basic case and one large scale case for solving the CHPED problem. The propose approach solves CHPED problems considering capacity limits and heat-power mutual dependency constraints. The MU algorithm was used in HDE to hand the equality and inequality constraints. The computation was implemented on a personal computer (Intel(R) Core(TM) i7-3770 CPU @ 3.4 GHz with 8G Ram) in FORTRAN-90 language. Setting factors used in this two cases are as follows; the population size N_p is set as 10 for HDE-MU. The iteration numbers of outer loop and inner loop are set to (outer, inner) as (30, 5000).

A. The basic case

A realistic CHPED system involves one conventional power unit, three cogeneration units, and one heat-only unit, are employed to be the basic case for comparing the proposed HDE-MU with previous methods in qualities and performances of the optimal solutions obtained. The system power demand (P_d) and heat demand (H_d) of this CHPED problem are 300MW and 150MWth, respectively. This case used the linear boundary of the Heat-Power feasible region for cogeneration units. The associated data of this CHPED problem are the same as [21].

The feasible power and heat output region of cogeneration unit 2, 3 and 4 are shown in Figs 2, 3 and 4, respectively. These Figs 2, 3 and 4 demonstrate correlations between the power and heat outputs.

The implementation of the proposed algorithm for this basic case can be described as follows:

$$L_a(x, v, v) = f(x) + \sum_{k=1}^2 \alpha_k \{ [h_k(x) + v_k]^2 - v_k^2 \} + \sum_{k=1}^8 \beta_k \{ \langle g_k(x) + v_k \rangle_+^2 - v_k^2 \} \quad (17)$$

$$\text{objective: } \min_{x=(P_1, P_2, P_3, P_4, H_2, H_3, H_4, H_5)} f(x)$$

$$f(x) = \text{Cost}_1(P_1) + \sum_{j=2}^4 \text{Cost}_j(H_j, P_j) + \text{Cost}_5(H_5) \quad (18)$$

where

$$\text{Cost}_1(P_1) = 0.000115P_1^3 + 0.00172P_1^2 + 7.6997P_1 + 254.8863$$

$$\text{Cost}_2(H_2, P_2) = 0.00435P_2^2 + 36P_2 + 0.027H_2^2 + 0.6H_2 + 0.011P_2H_2 + 1250$$

$$\text{Cost}_3(H_3, P_3) = 0.1035P_3^2 + 34.5P_3 + 0.025H_3^2 + 2.203H_3 + 0.051P_3H_3 + 2650$$

$$\text{Cost}_4(H_4, P_4) = 0.072P_4^2 + 20P_4 + 0.02H_4^2 + 2.3H_4 + 0.04P_4H_4 + 1565$$

$$\text{Cost}_5(H_5) = 0.038H_5^2 + 2.0109H_5 + 950$$

Subject to;

$$h_1: P_1 + P_2 + P_3 + P_4 - P_d = 0$$

$$h_2: H_2 + H_3 + H_4 + H_5 - H_d = 0$$

$$g_1: 1.15841584158H_2 - P_2 - 46.8811881188 \leq 0$$

$$g_2: \begin{cases} -P_2 + 44.0 \leq 0, & \text{if } H_2 < 15.9 \\ -0.0676818959H_2 - P_2 + 45.0761421319 \leq 0, & \text{if } H_2 \geq 15.9 \end{cases}$$

$$g_3: \begin{cases} P_2 - 125.8 \leq 0, & \text{if } H_2 < 32.4 \\ 0.1511627909H_2 + P_2 - 130.697674418 \leq 0, & \text{if } H_2 \geq 32.4 \end{cases}$$

$$g_4: -0.25H_3 - P_3 + 20 \leq 0$$

$$g_5: 7H_3 - 3P_3 - 250 \leq 0$$

$$g_6: 3H_3 + 11P_3 - 660 \leq 0$$

$$g_7: \begin{cases} -P_4 + 35.0 \leq 0, & \text{if } H_4 < 20.0 \\ 2.2H_4 - P_2 - 9 \leq 0, & \text{if } H_4 \geq 20.0 \end{cases}$$

$$g_8: \begin{cases} P_4 - 90.0 \leq 0, & \text{if } H_4 > 25.0 \\ 2.4H_4 + P_2 - 150 \leq 0, & \text{if } H_4 \leq 25.0 \end{cases}$$

TABLE1: COMPARES FIFTEEN COMPUTATIONAL RESULTS OBTAINED FROM THE PROPOSED HDE-MU AND PREVIOUS METHODS.

Items Methods	$P(G_1)$	$P(G_2)$	$P(G_3)$	$P(G_4)$	$H(G_2)$	$H(G_3)$	$H(G_4)$	$H(G_5)$	$\Sigma P(G)$	$\Sigma H(G)$	TC(\$/h)	Time (Sec)
HDE-MU	135.00000	40.74801	19.25199	105.00000	73.61350	36.72118	0.00000	39.66531	300.00000	150.00000	13672.83428	1.43
GAMS [10]	135.0000	23.3200*	10.9100	130.7640*	72.8370*	40.6000	0.0000*	36.5600	299.994	149.997	13605.37	N/A
CCM [11]	135.00	23.32*	10.92	130.76*	72.84*	40.60	0.00*	36.56	300.00	150.00	13605.40	N/A
HSGA [12]	134.86	23.84*	12.02	129.26*	74.88*	33.81	0.34*	40.97	299.98	150.00	13611.15	N/A
EDHS [13]	135.0000	18.1563*	13.0749	133.7688*	84.0626*	37.7657	0.0000*	28.1118	300.0000	149.9401	13613	N/A
PFCOA [14]	135.00	40.20	19.70	105.00	75.10	35.40	0.00	39.40	299.90	149.90	13668.40	27.06
GSA [15]	135.0000	41.7806	18.1736	105.0000	74.0890	37.3336	0.0000	38.5713	299.9542	149.9939	13671.1490	1.423
EMA [16]	135.0000	40.7163	19.2837	105.0000	73.7022	36.7183	0.0000	39.5829	300.0000	150.0034	13672.7407	N/A
BD [17]	135.0000	40.7687	19.2313	105.0000	73.5957	36.7759	0.0000	39.6284	300.0000	150.0000	13672.83	N/A
PSO-TVAC [18]	135.0000	41.4019	18.5981	105.0000	73.3562	37.4295	0.0000	39.2143	300.0000	150.0000	13672.8892	1.52
FA [19]	134.74	40.00	20.25	105.00	75.00	27.87	0.00	47.12	299.99	149.99	13683.22	N/A
IWO [20]	134.73	40.00	20.86	104.41	75.00	37.60	0.00	37.40	300.00	150.00	13683.56	N/A
CPSO [18]	135.0000	40.7309	19.2728	105.0000	64.4003	26.4119	0.0000	59.1955	300.0037	150.0076	13692.5212	1.50
HS [21]	134.74	48.20	16.23	100.85	81.09	23.92	6.29	38.70	300.02	150.00	13723.20	N/A
GA [21]	135.00	70.81	10.84	83.28	80.54	39.81	0.00	29.64	299.93	149.99	13779.50	N/A

NOTE: * DENOTING OPERATION IN THE INFEASIBLE REGION.

This minimum cost problem consists of one objective function with eight variable parameters, ($P_1, P_2, P_3, P_4, H_2, H_3, H_4$, and H_5), two equality constraints, (h_1 and h_2), and eight inequality constraints, ($g_1 \sim g_8$), in the Heat-Power available region.

This basic case shows the validity and practicability of general use for the CHPED problems. Table 1 compares fifteen computational results obtained from the proposed HDE-MU, generalized algebraic modeling system (GAMS) [10], canonical coordinates method (CCM) [11], harmony search with genetic algorithm (HSGA) [12], harmony search for economic dispatch (EDHS) [13], cuckoo optimization algorithm with penalty function (PFCOA) [14], gravitational search algorithm (GSA) [15], exchange market algorithm (EMA) [16], benders decomposition (BD) [17], particle swarm optimization with time varying acceleration coefficients (TVAC-PSO) [18], firefly algorithm (FA) [19], invasive weed optimization (IWO) [20], classic PSO (CPSO) [19], harmony search (HS) [21] and genetic algorithm (GA) [21]. Although results obtained from GAMS [10], CCM [11], HSGA [12] and EDHS [13] have the less cost than the proposed HDE-MU, the bookmarks indicate that their outputs of (H_2, P_2) and (H_4, P_4) are invade infeasible heat-power regions. Hence, the following works are not considered competitive: GAMS [10], CCM [11], HSGA [12] and EDHS [13]. The heat outputs and/or power outputs of PFCOA [14], GSA [15], EMA [16], FA [19], IWO [20], CPSO [18], HS [21] and GA [21] didn't match the heat demand ($H_d = 150\text{MWth}$) and power demand ($P_d = 300\text{MW}$) that will cause an unbalance of system's heat and power demands. By investigating the results presented in Table 1, it is observed that the best total cost utilizing HDE-MU is 13,672.83428 \$/h, which is much less than the best results previously reported in PSO-TVAC [18] and IWO [20], and very close to the best result of BD [17]. As seen from the best solution of HDE-MU listed in Table 1, the power output is 300.00000 MW and the heat output is 150.00000 MWth.

Apparently, the equality constraints (5) and (6) of power and heat demands are fully satisfied. Therefore, the result obtained from the proposed HDE-MU is an optimal and feasible solution. According to the absolute CPU time of different methods listed in the Table 1, the proposed HDE-MU is faster than most of the compared algorithms, but slightly slower than GSA [15]. Consequently, the HDE-MU is not only supplying the exact total of heat and power, but also has the less CPU time than most approaches for the cogeneration system. Table 1 reveals that the HDE-MU is a satisfactory algorithm for solving the CHPED problem. The convergence characteristics of the proposed method in comparison with DE with MU (DE-MU) for this real example are depicted in Fig. 5, and the distribution of the minimum costs after 100 trials by the HDE-MU is plotted

TABLE 2: COMPUTATIONAL RESULTS OF THE DE-MU AND THE PROPOSED HDE-MU FOR THE LARGE SCALE CHPED PROBLEM.

Methods	DE-MU, ($N_p=30$)					HDE-MU, ($N_p=10$)				
No. of units	20	50	100	200	400	20	50	100	200	400
Cost_av (\$/h)	54,732.14	136,837	274,159	555,212	1,294,645	54,692.05	136,732	273,481	547,121	1,095,934
CPU_av (sec)	17.85	42.63	88.42	193.93	435.71	5.74	14.36	28.95	58.17	122.28
#ΔCost_av(%)	0.0013	0.0027	0.0089	0.0380	0.1927	-	-	-	-	-

$$\# \Delta \text{Cost_av}(\%) = \frac{\text{Cost_av of the DE-MU} - \text{Cost_av of the proposed HDE-MU}}{\text{Cost_av of the proposed HDE-MU}} \times 100\%$$

in Fig. 6. It can be observed that the worst, average and best costs are very close. The small standard deviation again confirms the stability of the proposed HDE-MU.

B. The large scale case

This test shows that the proposed approach is also applicable to the large scale system of the actual CHPED problem. The basic case given above is expanded to build five systems with 20, 50, 100, 200 and 400 units. To avoid confusion owing to the stochastic nature of the solution methods, computational results of 50 runs beginning with random initial populations were averaged, and are given in Table 2. This table reveals that the proposed approach is superior to the DE-MU in terms of average cost (Cost_av) and average CPU time (CPU_av) in all cases of units. The advantage of the proposed algorithm is mostly in the solution quality and calculation speed, and its superiority becomes clearer as the system size is enlarged. Moreover, the proposed method requires a smaller population size than the DE-MU. Significantly, the computational burden of the proposed algorithm grows almost linearly with respect to the increase in system size, showing that proposed approach is applicable to large scale CHPED problems of power systems.

V. CONCLUSIONS

The HDE-MU for solving the CHPED problem has been proposed herein. Complication of the CHPED problem lies in the constraints imposed by the mutual dependencies of multi-demand and Heat-Power capacity. The HDE helps the proposed method efficiently search and refined exploit. The MU helps the proposed method avoid deforming the ALF and resulting in difficulty of solution searching. The proposed algorithm integrates the HDE and the MU that has the merits of taking a wide range of penalty parameters and a small population. A practical CHPED system is used to compare the proposed HDE-MU with fifteen reported methods. Simulation results demonstrate that the proposed algorithm has more advantages for solving the CHPED problem than the previous methods. The contributions of this paper are the MU effectively handles the Heat-Power feasible region constraints, the HDE efficiently searches the optimal solutions in the economic dispatch process, and the author provides a valid and efficacious algorithm for the large scale CHPED problem of power systems.

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