

Sensitivity Base Approach for the Optimal Sizing and Allocation of Distributed Generation in a Radial Network

Paul C. Maduforo, Jonas N. Emechebe, Emmanuel M. Eronu, and Stephen A. Benson

Abstract—Electric power has remained a basic essential for the advancement of any nation's economy. Increasing human exercises because of innovative advancement combined with populace development has made the interest for power dramatically increasing continuously, subsequently widening the gap between power generated and the demand by the consumers. This work provides sensitivity based method for the allocation of a distributed generation in a distribution network aimed at improving the voltage profile and reduce power loss in the quest to narrow the gap between power generated and that demanded by the consumers. Using the Loss sensitivity method, a DG of 153kW was allocated to bus 5 that sees a power reduction of 46% with voltage profile improved within constraints. Voltage sensitivity index was calculated at all nodes. Bus 17 was found to have the minimum VSI. In this case DG sizes were taken in step size of 17.5kW starting from 30 kW till 170 kW at different power factors of 1.0, 0.9, 0.85, and 0.8. The DG sizes were tested at the selected power for various DG sizes. 135kW DG at unity power factor was installed. After comparing the two methods it can be concluded that loss reduction in loss sensitivity method is more and it is better in terms of selecting the optimal location for the placement of DG. For the purpose of sizing the voltage sensitivity analysis index method is a better option.

Index Terms—Distributed Generation, Radial Network, Loss Sensitivity Base, Power Loss, Voltage.

I. INTRODUCTION

With the increase in power utilization, realizing the power request of customers at all situation inside the power network economically and reliably as possible has become a definitive point of the power system. The customary practice of the electric electricity generation system uses the orthodox energy sources amongst which in Nigeria generally used are hydro and thermal with continuing advances in solar [1]. More so, to improve the performance of the surviving distribution network, DG offers a feasible means of curbing to some extent some circumstances faced in terms of power losses and voltage profile improvement. Distribution systems are either radial or feebly meshed generally, because of high R/X ratio causing more power

losses and voltage drop. Power loss minimization plays a crucial role for economic operation and energy cost reduction. There are numerous approaches to lessen the losses like capacitor assignment, Distributed Generation placement, load management and system reconfiguration [2]. Hence, this work provides a sensitivity based method for the allocation of a distributed generation in a distribution network aimed at improving the voltage profile and reduction of power loss in the quest to narrow the gap between power generated and that demanded by the consumers.

II. LITERATURE REVIEW

Distributed generation (DG) is a rising methodology for providing electric power near the load centers. It involves the installation and operation of an arrangement of little size, conservative and clean electric power generating units at or near the electric load [3]. The term Distributed Generation for the most part, alludes to small scale (customarily 1 KW–50 MW) electric power generators that produce power at a site near the consumers or that are attached to an electric distribution system. DGs incorporate synchronous generators, induction generators, reciprocating engines, micro grid turbines, combustion gas turbines, fuel cells, solar photovoltaic, wind turbines and other small power sources. DG can be used to generate a customer's entire electricity supply for peak shaving or standby or emergency generation, as a green power source or for increased reliability. DGs can be less exorbitant as it eliminates the requirement for costly construction of distribution and transmission lines. All the more thus, DGs can give economically effective; natural well disposed, high power quality and more reliable energy arrangements than ordinary age [4]

Several techniques recently have been suggested for the optimal placement of DG in a distribution system for power loss minimization. These techniques involve analytical method, Genetic Algorithm, Particle Swarm Optimization, Harmony Search Algorithm, Simulated Annealing (SA). Moradi et al. [5] presented a new combined method to solve siting and sizing problems for DG and capacitor banks simultaneously in distribution system. The ICA algorithm was used to find location and size of the DGs and the capacitors. Then GA was used to generate a new set of colonies and solutions in the all search spaces. Combined method was implemented to minimize the losses, to increase the voltage stability, to improve the voltage regulation index and to balance the loads. Khatod et al. [6] developed an EP based technique for optimal placement of DG units energized by renewable energy resources in a radial

Published on June 30, 2020.

P. C. Maduforo, Department of Electrical Electronic Engineering, University of Abuja, Nigeria.

(e-mail: gdonrpm@gmail.com)

J. N. Emechebe, Federal Radio Corporation of Nigeria, Abuja F.C.T, Nigeria.

(e-mail: jonasemechebe@gmail.com)

E. M Eronu, Department of Electrical Electronic Engineering, University of Abuja, Nigeria.

(e-mail: eronu.majjiyebo@uniabuja.edu.ng)

S. A. Benson, Department of Electrical Electronic Engineering, University of Abuja, Nigeria.

(e-mail: iamsteveben@gmail.com)

distribution system. For tackling uncertainties related with load and renewable resources, probabilistic techniques had been used. For the approach, an index based scheme had also been developed to generate the population ensuring the feasibility of each individual and thus considerably reducing the computational time.

Garcia and Mena [7] used a new evolutionary method called Teaching–Learning Based Optimization algorithm to find the best sites to connect DG systems in a distribution network, choosing among a large number of potential combinations. Georgilakis et al. [8] aimed at providing the best site and sizes of DGs to optimize radial distribution network operation and planning taking into account DG capacity constraints. Various models and methods had been suggested for the solution of the ODGP problem. This paper presented an overview of the state of the art models and methods applied to the ODGP problem, analyzing and classifying current and future research trends in this area. Shukla et al. [9] minimized active power loss by placing DG in radial distribution system. The problem was formulated as optimization problem and solution was obtained using genetic algorithm. The locations were decided on the basis of loss sensitivity to active power injection at various nodes. The performance of the method was tested on 33-bus test system and comparison of the results with a reported method revealed that the method yielded workable results.

III. MATERIAL AND METHOD

The data and network diagram for this research is obtained from Abuja Electricity Distribution Company. Fig. 1 shows the 11kV feeder one of the Gwagwalada injection sub-station, Abuja Nigeria was highlighted and used to demonstrate the use of the stated technique for the sizing and positioning of the DG.

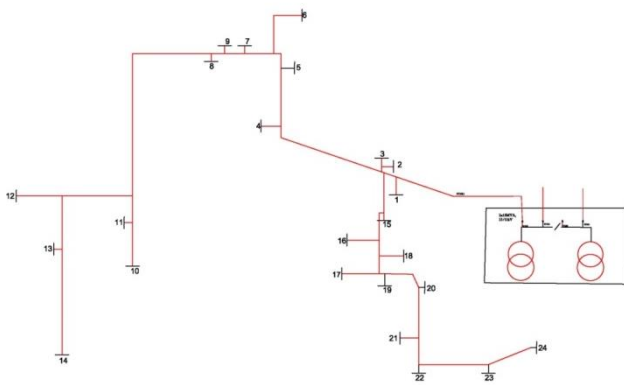


Fig. 1. 24 Bus radial distribution network of Gwagwalada injection sub-station feeder 1

A. Problem Formulation

The power flow problem is a very familiar problem in the field of power systems engineering, where voltage magnitudes and angles for one set of buses are sought after, given that voltage magnitudes and power levels for another set of buses are known and that a model of the network configuration is accessible. A power flow solution procedure is a numerical technique that is employed to resolve the power flow problem [10].

1) Real and Reactive Power Injected in a Bus

An electric power system is fundamentally made up of generators, transformers, transmission lines and loads. A modest power system is exemplified in Fig. 2. Hence, the network formed by these static components can be seen or taken as a linear network and represented by the matching admittance matrix or impedance matrix. In power flow calculation, the generators and loads are treated as nonlinear components and cannot be embodied in the network.

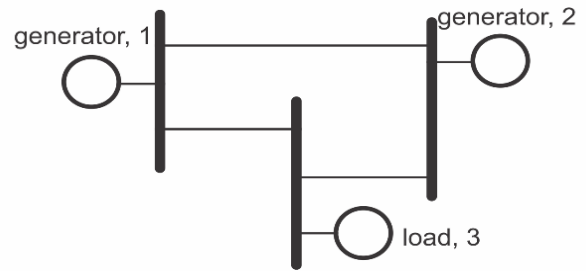


Fig. 2a. Single-line diagram of a simple network

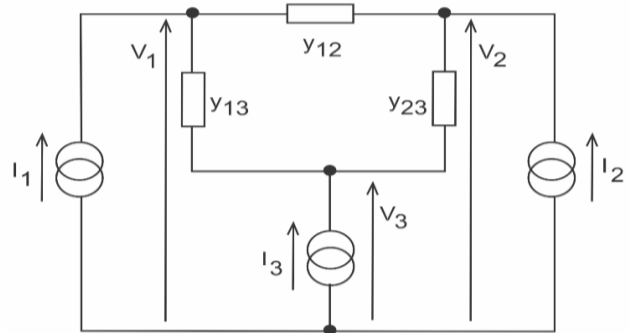


Fig. 2b. Equivalent circuit for one phase power system of the system shown in Fig 2a

In formulating the real and reactive power entering a bus, we need to describe the following variables. Take the *i*th bus voltage to be given as:

$$V_i = |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i) \tag{1}$$

And self-admittance at the bus-*i* as:

$$Y_{ii} = |Y_{ii}| \angle \theta_{ii} = |Y_{ii}| (\cos \theta_{ii} + j \sin \theta_{ii}) = G_{ii} + jB_{ii} \tag{2}$$

Likewise, the mutual admittance between the buses *i* and *j* can be written as:

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| (\cos \theta_{ij} + j \sin \theta_{ij}) = G_{ij} + jB_{ij} \tag{3}$$

Supposing an absolute number of *n* buses is enclosed in the power system. The infused or injected current at *i*-bus is assumed as,

$$\begin{aligned} I_i &= Y_{i1}V_1 + Y_{i2}V_2 + \dots + Y_{in}V_n \\ &= \sum_{k=1}^n Y_{ik}V_k \end{aligned} \tag{4}$$

It is to be well-known; we will expect the current entering a

bus to be positive and that leaving the bus to be negative. As a result, the reactive and real power entering a bus will also be presumed as to be positive. The intricate power at bus i is then set by

$$\begin{aligned}
 P_i - jQ_i &= V_i^* I_i = V_i^* \sum_{k=1}^n Y_{ik} V_k \\
 &= |V_i| (\cos \delta_i - j \sin \delta_i) \sum_{k=1}^n |Y_{ik} V_k| (\cos \theta_{ik} + j \sin \theta_{ik}) (\cos \delta_k + j \sin \delta_k) \\
 &= \sum_{k=1}^n |Y_{ik} V_i V_k| (\cos \delta_i - j \sin \delta_i) (\cos \theta_{ik} + j \sin \theta_{ik}) (\cos \delta_k + j \sin \delta_k)
 \end{aligned} \tag{5}$$

Note that,

$$\begin{aligned}
 &(\cos \delta_i - j \sin \delta_i) (\cos \theta_{ik} + j \sin \theta_{ik}) (\cos \delta_k + j \sin \delta_k) \\
 &= (\cos \delta_i - j \sin \delta_i) [\cos(\theta_{ik} + \delta_k) + j \sin(\theta_{ik} + \delta_k)] \\
 &= \cos(\theta_{ik} + \delta_k - \delta_i) + j \sin(\theta_{ik} + \delta_k - \delta_i)
 \end{aligned} \tag{6}$$

Hence, rearranging (5) and substituting, (4) gives the real and reactive power as

$$P_i = \sum_{k=1}^n |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \tag{7}$$

$$Q_i = - \sum_{k=1}^n |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \tag{8}$$

2) Preparation of Data for Load Flow

Let real and reactive power produced at bus-i be represented by P_{Gi} and Q_{Gi} separately. Additionally, let us represent the reactive and real power expended at the ith bus by Q_{Li} and P_{Li} separately. At that point net real power infused or injected in bus-i is

$$P_{i,inj} = P_{Gi} - P_{Li} \tag{9}$$

Let the infused power determined by the load flow program be $P_{i,calc}$. At that point the mismatch between the actual infused and determined values is specified by,

$$\Delta P_i = P_{i,inj} - P_{i,calc} = P_{Gi} - P_{Li} - P_{i,calc} \tag{10}$$

Along these lines the mismatch between the reactive power infused and determined values is set as

$$\Delta Q_i = Q_{i,inj} - Q_{i,calc} = Q_{Gi} - Q_{Li} - Q_{i,calc} \tag{11}$$

Also, the objective of the load flow is to minimize the above two mismatches defined in equation (10) and (11). However, since the magnitudes or levels of all the voltages and their angles are not given or known, an iterative process must be used to approximate the bus voltages and their angles in order to compute the mismatches. It is anticipated that mismatches P_i and Q_i diminish with progressive iteration and the load flow stream is said to have merged or converged when the mismatches of the considerable number of buses become not exactly an exceptionally modest number (i.e. small number).

Upon completion of the load flow analysis of the network using MatPower, the outcomes at the various buses were observed and recorded in Table I which shows the network voltages level and real power losses at each bus. Thus, knowing the state of the network provides a clear pattern to understand the behavior of the system and to know the bus that have challenge of poor voltage profile and power loss to enable the process of improving the stability of the system; since the utmost aim of the power system operation is to ensure continuity in supply.

TABLE I: INITIAL VOLTAGE PROFILE AND REAL POWER AT THE BUSES.

BUS #	VOLTAGE IN P.U	POWER LOSS (kW)
1	1.0000	0.0522
2	0.9700	0.0100
3	0.9613	0.7124
4	0.9711	2.4239
5	0.9613	0.4695
6	0.9603	0.2836
7	0.9558	0.3950
8	0.9428	0.5899
9	0.9537	0.6310
10	0.9367	1.7508
11	0.9319	0.1111
12	0.9296	0.0098
13	0.9284	3.7309
14	0.9287	1.4021
15	0.9370	0.5292
16	0.9349	0.2221
17	0.9360	0.0072
18	0.9586	2.4811
19	0.9544	0.0049
20	0.9563	0.0193
21	0.9549	4.9225
22	0.9525	0.7285
23	0.9584	1.4613
24	0.9514	3.0690

B. Methods Implemented

Numerous techniques are available for sizing and location of DG in radial distribution networks. Out of those several techniques two techniques this work considered the loss sensitivity analysis and compares the results with the voltage sensitivity analysis technique and the results compared.

1) Loss Sensitivity Analysis

Sensitivity factor technique is centered on the principle of linearization of original nonlinear equation around the introductory working point, which assists with diminishing number of solution space. Loss sensitivity factor technique is used to take care of the capacitor placement issue. Its application in DG area is new in this field and has been detailed [11]. The exact loss formula is used to ascertain the real power loss in the system. The real power loss sensitivity factor with respect to real power injection is achieved by differentiating exact loss formula with regard to real power injection at bus P_i which is given by:

$$\alpha_i = \frac{\delta P_i}{\delta P_i} = 2 \sum_{j=1}^N (\alpha_{ij} P_j - \beta_{ij} Q_j) = 0 \quad (12)$$

This follows that,

$$P_i = \frac{1}{\alpha_{ii}} [\beta_{ii} Q_i + \sum_{j=1, j \neq i}^N (\alpha_{ij} P_j - \beta_{ij} Q_j)] \quad (13)$$

where P_i denotes the real power infused or injection at node i , which is the contrast between real power generation and real power demand or request at that node.

$$P_i = P_{DG_i} - P_{D_i} \quad (14)$$

where P_{DG_i} is the real power infusion or injection from DG located at node i , P_{D_i} is the load request or demand at node i , relating (12) & (13) we get

$$P_{DG_i} = P_{D_i} + \frac{1}{\alpha_{ii}} [\beta_{ii} Q_i + \sum_{j=1, j \neq i}^N (\alpha_{ij} P_j - \beta_{ij} Q_j)] \quad (15)$$

The above equation decides the size of the DG at which the losses are least. By ordering the list in ascending direction, the bus remained in the top is ranked or position as the first location or position of DG and further the procedure is repeated by insertion the concerned size of DG at that specific location which produces the next location of DG. The procedure is said to be ended when it decides the same position or location.

2) Bus Voltage Sensitivity Analysis

Alternative technique for minimizing the search space is the voltage sensitivity analysis at the bus. For this situation each bus is infiltrated at a time, by a DG of 20% size of the maximum feeder loading capacity. Subsequent to placing DG at every node its voltage sensitivity index can be determined by equation (15). At the point when DG is coupled at bus i , voltage sensitivity index for bus i is specified by:

$$BVS_i = \frac{\sqrt{(\sum_{k=1}^N (1 - V_k)^2)}}{\sqrt{N}} \quad (16)$$

where V_k is the voltage at k^{th} node and N is the total number of nodes. The node with the smallest BVS_{*i*} will be selected for DG allocation. The algorithm for DG location and sizing can be given as:

- Step 1: Obtain the load flow for the base case.
- Step 2: Using equation (15) find the Bus voltage sensitivity indices at the respective node by penetrating the 20 % of DG value at respective node and rank the sensitivities of all nodes in a rising or ascending manner to form priority list.
- Step 3: Pick the bus with least priority and place DG at that bus.
- Step 4: Change the size of DG in small steps and calculate power loss for each by running load flow.
- Step 5: Store the size of DG that gives minimum or least loss.
- Step 6: Compare the loss with the preceding solution. In the event that the loss is less than preceding previous solution, store this new solution and Discard previous solution.

Step 7: Repeat Step 4 to Step 6 for all buses in the priority list.

C. Distributed Generation Sizing

Typically, the essence for the inclusion of any power source into an existing network is to compliment the amount of energy in the power system. This is often done also to make up for the losses the power system or network may be experiencing at any given instant at each bus. The size of DG in any network is limited and falls between the ranges of 15% to 30% of total load taken at the individual buses of which for the network under study, using the specified range falls between 20kw and 138kw. It is assumed that the specified range if accounted for will likely represent the Power losses at the bus.

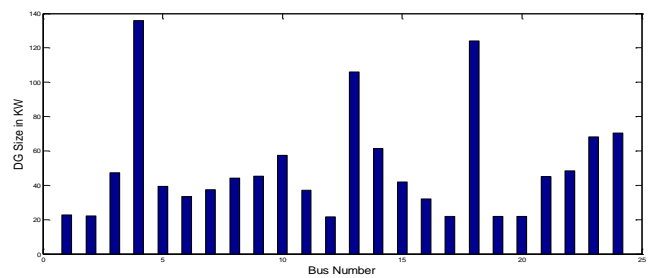


Fig. 3. Distribution Generation sizes at various Buses

IV. RESULTS AND DISCUSSION

A. Loss Sensitivity Analysis Method

Loss sensitivities were evaluated at all the buses. The bus with smallest sensitivity was selected as optimal location which happens to be bus 5 of the tested radial network. Distributed generator size was tested in the range of 30kW to 170kW with the step size of 17.5kW. Optimal size was obtained as 153 kW. A loss reduction of approximately 46% was obtained using this method. The voltage profiles using this technique are given in Table II within voltage constraint which shows the network without DG and with DG placement.

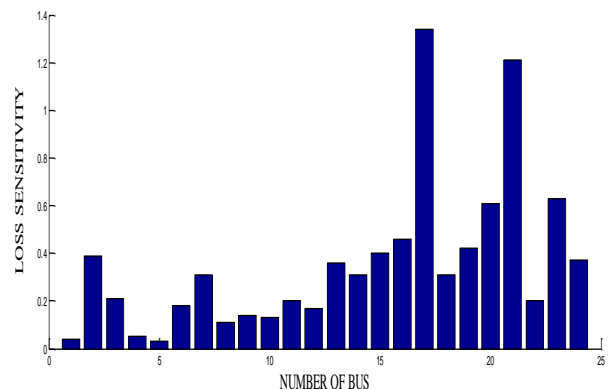


Fig. 4. Loss sensitivity factor at the buses

TABLE II: VOLTAGE PROFILES WITHOUT AND WITH DG

BUS NUMBER	WITHOUT DG	WITH DG
1	1	1
2	0.99703	0.99854
3	0.98289	0.99252
4	0.97538	0.99101
5	0.96796	0.98985
6	0.94948	0.985
7	0.94595	0.98162
8	0.9323	0.96849
9	0.92597	0.9624
10	0.92011	0.95677
11	0.91924	0.95594
12	0.91773	0.95449
13	0.91155	0.94855
14	0.90926	0.94636
15	0.90784	0.94498
16	0.90645	0.94366
17	0.90441	0.94169
18	0.90379	0.9411
19	0.9965	0.99801
20	0.99292	0.99444
21	0.99221	0.99374
22	0.99158	0.9931
23	0.97931	0.98897
24	0.97264	0.98236

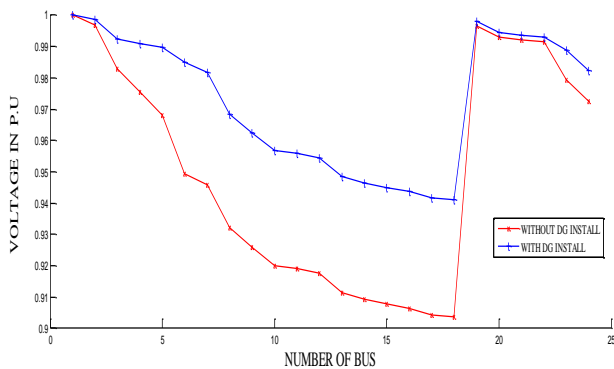


Fig. 4. Voltage Profile in using Loss Sensitivity Method

The real power loss at each bus shows the result given in Fig. 5.

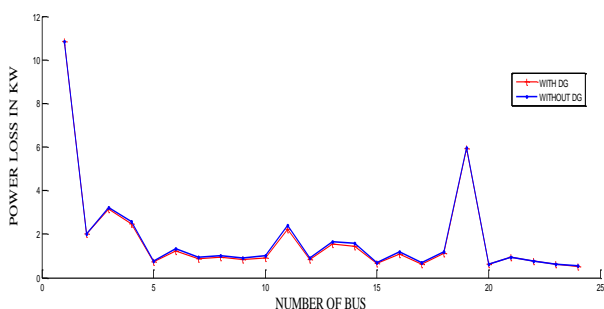


Fig. 5. Power Loss at the bus

B. Voltage Sensitivity Index Method

In this technique voltage sensitivity index was calculated at all nodes. Bus 17 was found to have the minimum VSI. In this case DG sizes were taken in step size of 17.5kW starting from 30 kW till 152.5 kW at different power factors of 1.0, 0.9, 0.85, and 0.8. The DG sizes were tested at the selected power for various DG sizes as shown in Table III. Hence the selected optimal DG placed at this bus 135kW at unity power factor with real power loss as indicated in Table III.

TABLE III: DG SIZES TESTED IN VSI METHOD

DG SIZE IN Kw with Power Factor			
U pf	0.9 pf	0.85 pf	0.8 pf
30	28	25	23
47.5	45	40	38
65	63	55	49
82.5	80	75	67
100	95	88	70
117.5	110	105	90
135	125	120	107
152.5	149	138	128

The result of the calculated voltage sensitivity indices at the individual buses is given Fig. 6.

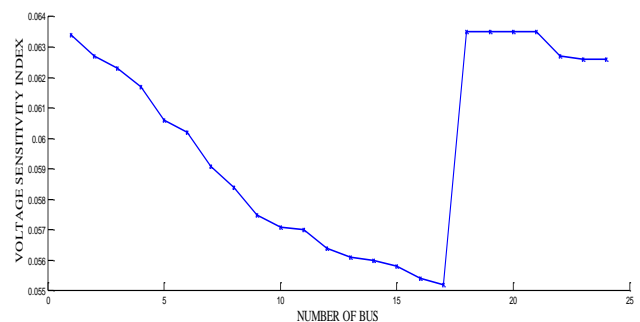


Fig. 6. Voltage sensitivity at the various Buses

The voltage sensitivities and DG sizes tested have been shown above. The total power loss reduction of the network ranges from 28-33 %. Taking the DG range considered, the respective cumulative real power losses are given in Fig. 7 with the power factor of 0.8 have better reduced losses.

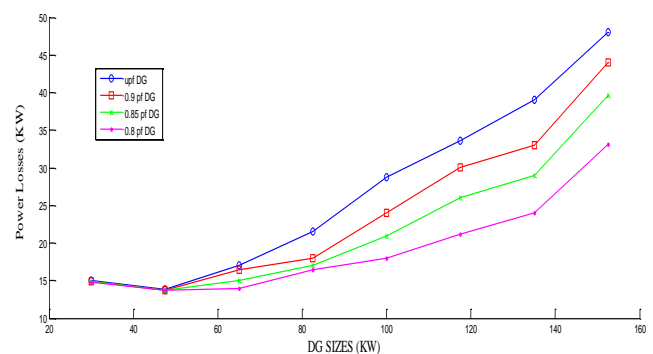


Fig. 7. Power Loss Curve

Table IV shows the voltage profiles of the DG at the selected power factor upon installation of DG of 135kW

TABLE IV: VOLTAGE PROFILE USING VSI METHOD

Bus	Without DG	Unity PF	0.9 PF	0.85 PF	0.8PF
1	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.99703	0.99765	0.99759	0.99756	0.99756
3	0.98289	0.98685	0.98650	0.98632	0.98632
4	0.97538	0.98181	0.98123	0.98094	0.98094
5	0.96796	0.97696	0.97615	0.97574	0.97574
6	0.94948	0.96407	0.96277	0.96211	0.96211
7	0.94595	0.96181	0.96041	0.95969	0.95969
8	0.93230	0.95945	0.95703	0.95580	0.95580
9	0.92597	0.95987	0.95685	0.95531	0.95531
10	0.92011	0.96083	0.95719	0.95534	0.95534
11	0.91924	0.96125	0.95749	0.95558	0.95558
12	0.91773	0.96218	0.95820	0.95617	0.95617
13	0.91155	0.96557	0.96071	0.95825	0.95825
14	0.90926	0.96679	0.96162	0.95899	0.95899
15	0.90784	0.96916	0.96364	0.96084	0.96084
16	0.90645	0.97258	0.96661	0.96358	0.96358
17	0.90441	0.97879	0.97206	0.96865	0.96865
18	0.90379	0.98286	0.97570	0.97207	0.97207
19	0.99650	0.99712	0.99706	0.99704	0.99704
20	0.99292	0.99355	0.99349	0.99346	0.99346
21	0.99221	0.99284	0.99278	0.99276	0.99276
22	0.99158	0.99220	0.99215	0.99212	0.99212
23	0.97931	0.98328	0.98292	0.98274	0.98274
24	0.97264	0.97664	0.97628	0.97610	0.97610

Fig. 8 depicts the graphical representation of the Table IV.

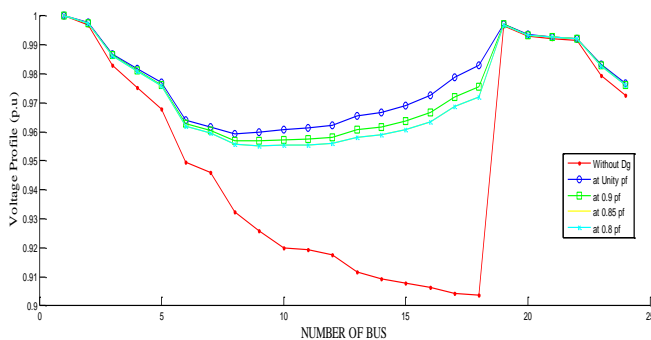


Fig. 8. Voltage profile using VSI method

After comparing the two methods it can be concluded that loss reduction in loss sensitivity method is more and it is better in terms of selecting the optimal location for the placement of DG. For the purpose of sizing the voltage sensitivity analysis index method is a better option.

V. CONCLUSION

This work presented the formulation and application of sensitivity based method to support in improvement of voltage profile with consequent reduction in power loss by optimizing the size and location of a DG. Following the results obtained using the sensitivity methods; it becomes obvious that a better loss reduction was obtained in loss sensitivity method where bus 5 was highlighted for DG allocation since it was a bus least loss sensitivity and result in improved voltage profile with a 46% reduction in power loss when a DG of 153kW was installed. Implementing voltage sensitivity index method, though loss reduction was minimal, but the calculation of distributed generation sizing was better. Meanwhile voltage sensitivity index method largely provides minimum sensitivity values at last buses; therefore it is a very unyielding and inappropriate method for loss reduction and voltage profile improvement.

REFERENCES

- [1] E. C. Ashigwuike, and S. A. Benson, 'Optimal Location and Sizing of Distributed Generation in Distribution Network Using Adaptive Neuro-Fuzzy Logic Technique', Vol. 4, No. 4, PP.83-89 April 2019
- [2] P.S Georgilakis, N.D. Hatzigiargyriou, 'Optimal Distributed Generation Placement in Power Distribution Networks: Models, Methods, and Future Research', IEEE Transactions on Power Systems, Vol. 28, No. 3, pp. 3420-3428, 2013.
- [3] W. El-Khattam, M.M.A. Salama, "Distributed generation technologies, definitions and benefits", Electric Power System Research, Vol 71, pp. 119-128, 2004.
- [4] P.V. Babua, S.P. Singhb, "Optimal Placement of DG in Distribution network for Power loss minimization using NLP & PLS Technique", 5th International Conference on Advances in Energy Research, ICAER 2015 December 2015, Mumbai, India, pp. 15-17.
- [5] M. H. Moradi and M. Abedini, "A combination of GA and PSO for optimal location and sizing of DG", Electrical Power and Energy Systems- Elsevier, Vol. 34, No. 1, pp. 66-74, 2012.
- [6] D. K. Khatod, V. Pant, J. D. Sharma "Evolutionary Programming Based optimal Placement of renewable distributed generators" IEEE Transactions on Power Systems, Vol.28, No. 4, pp. 683-695, 2013.
- [7] J. A. M. Garcia and A. J. G. Mena "Optimal distributed generation location and size using a modified teaching-learning based optimization algorithm" Electric Power and Energy System, Elsevier, Vol.50, No.1, pp. 65-75, 2013.
- [8] P. S. Georgilakis and N. D. Hatzigiargyriou, "Optimal DG placement in power distribution networks: Models, Methods and Future research", IEEE Transactions on Power Systems, Vol. 28, No. 3, pp. 3420-3428, 2013.
- [9] T. N. Shukla, S. P. Singh and K. B. Naik, "Allocation of optimal DG using GA for minimum power losses in distribution systems", International Journal of Engineering Sciences and Technology, Vol. 2, No. 3, pp. 94-106, 2010.
- [10] S. Ghosh, S. P. Ghoshal and S. Ghosh, "Optimal sizing and placement of distributed generation in a network system", Electrical Power and Energy Systems- Elsevier, Vol. 32, No. 1, pp. 849-856, 2010.
- [11] Diagram of an Electric Power System, available at: http://en.wikipedia.org/wiki/Electric_power_transmission.