Introduction into the Extra Geometry of the Three-Dimensional Space II

I. Szalay and B. Szalay

***Abstract*—Using the theory of exploded numbers by the axiom – systems of real numbers and Euclidean geometry, we introduce concept of extra - plane of the three – dimensional space. The extra-planes are visible subsets of super– planes which are exploded Euclidean planes. We investigate the main properties of extra-planes. We prove more similar properties of Euclidean planes and extra–planes, but with respect to the parallelism there is an essential difference among them.**

***Index Terms*—Exploded and Compressed Numbers, Super Plane, Extra Plane, Border Lines, Extra Parallelism**

I. Introduction

We imagine our universe as the familiar three dimensional Euclidean space

$$\mathbb{R}^3 = \left\{ P = (x,y,z) \,\middle|\, \begin{Bmatrix} -\infty < x < \infty \\ -\infty < y < \infty \\ -\infty < z < \infty \end{Bmatrix} \right\},$$

with its well-known apparatus of the ordered field $(\mathbb{R}, \leq, +, \cdot)$ of real numbers and the vector-algebra of $\mathbb{R}^3$. Apparatus of exploded and compressed numbers (see [1], Chapter 2) is used, too, especially important is the concept of box-phenomenon. (See [1], Chapter 6. point 6.3 or [2], (8).) Other postulates, requirements, definitions, identities of explosion and compression are collected in the Part I of [2]. The idea of extra–line and extra-plane was alrady introduced in [1]. (See [1], Chapter 7, points 7.1 and 7.2). An important characterization of the extra-line was mentioned earlier (See [3], Theorem 1.10.) Moreover [2] contains the characterization of extra – lines and the extra - collinearity (see Properties 1, 2 and 3) and the main results for extra – parallelism of extra–lines. (See the Properties 4, 5 and 6.) The present article is a continuation of the foundation of extra geometry so, we continue Parts I, II and III with formulas (1) - (38), Fig. 1-3, Properties 1-6, Examples 1 and 1* and Definition 1 (concept of extra parallelism for extra-lines) to be found in the [2].

II. The Characterization of Extra Planes

The points $P = (x,y,z)$ of the Euclidean planes $\mathbb{S}_{P_0;N}$ are described by the vector-equation

$$P = P_0 + t \cdot E + s \cdot F \quad , (t,s) \in \mathbb{R}^2 , \tag{1}$$

where the point $P_0 = (x_0, y_0, z_0) \in \mathbb{R}^3$ and vectors $E = (e_x, e_y, e_z)$ and $F = (f_x, f_y, f_z) \in \mathbb{R}^3$ are given such that their norm $\|E\| = 1 = \|F\|$ and inner product $E \cdot F = 0$. The vectorial product

$N = (e_y f_z - e_z f_x , e_z f_x - e_x f_z , e_x f_y - e_y f_x)$.

Clearly, $\|N\|$=1. Having that $N \cdot E = 0 = N \cdot F$ the equation (1) yields

$$N \cdot P = N \cdot P_0 . \tag{2}$$

Hence, we get another equation for

$$\mathbb{S}_{P_0;N} n_x \cdot x + n_y \cdot y + n_z \cdot z = n_x \cdot x_0 + n_y \cdot y_0 + n_z \cdot z_0 \tag{3}$$

where $n_x = e_y f_z - e_z f_x$, $n_y = e_z f_x - e_x f_z$ and $n_z = e_x f_y - e_y f_x$.

Finally, by (1) we can write

$$\mathbb{S}_{P_0;N} = \left\{ (x,y,z) \in \mathbb{R}^3 \,\middle|\, \begin{cases} x = x_0 + te_x + sf_x \\ y = y_0 + te_y + sf_y \\ z = z_0 + te_z + sf_z \end{cases} ; (t,s) \in \mathbb{R}^2 ; N = E \times F \text{ (vectorial product)} \right\} \tag{4}$$

Similarly to the Parts I, II and III, we use the parameter $\sigma = 1$, so the explosion and compression is denoted without σ. The set $\widetilde{\mathbb{S}_{P_0;N}}$ is called super–plane. (See (5) in Part I of [2]) The super–planes are situated in the Multiverse. Denoting that $\mathcal{P} = \check{P}$ and using that $u = \check{x}, v = \check{y}, w = \check{z}$, by (41) we can write

$$\widetilde{\mathbb{S}_{P_0;N}} = \{ \mathcal{P} = (u,v,w) \in \widetilde{\mathbb{R}^3} \,|\, n_x \cdot \underline{u} + n_y \cdot \underline{v} + n_z \cdot \underline{w} = n_x \cdot x_0 + n_y \cdot y_0 + n_z \cdot z_0 \} \tag{5}$$

where $P_0 = (x_0, y_0, z_0) \in \mathbb{R}^3$ is a given point and the (normal vector) $N = (n_x, n_y, n_z) \in \mathbb{R}^3$ with $\|N\| = 1$. Clearly, $\widetilde{P_0} = (\widetilde{x_0}, \widetilde{y_0}, \widetilde{z_0}) \in \widetilde{\mathbb{S}_{P_0;N}}$.

It is known that the equation

$$A \cdot x + B \cdot y + C \cdot z = D \quad ; \quad A,B,C,D \in \mathbb{R} \text{ and } A = B = C = 0 \text{ is not allowed} \tag{6}$$

represents an Euclidean plane. So, the equation

$$(\mathcal{A} \,\overline{\odot}\, u) \,\overline{\oplus}\, (\mathcal{B} \,\overline{\odot}\, v) \,\overline{\oplus}\, (\mathcal{C} \,\overline{\odot}\, w) = \mathcal{D} \tag{7}$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} \in \widetilde{\mathbb{R}}$ *and* $\mathcal{A} = \mathcal{B} = \mathcal{C} = 0$ is not allowed, represents a super-plane. (Super-operations were defined in

---

Published on August 26, 2020.
I. Szalay and B. Szalay, University of Szeged, Hungary.

Part I. See Postulates of super-addition and super-multiplication.)

If $P_0 \in \underline{\mathbb{R}}^3$ $(\Leftrightarrow \widetilde{P_0} \in \mathbb{R}^3$, see (7) in Part I of [2].$)$ then the joint part of the Euclidean plane represented by (6) and the closed cube $\overline{(\mathbb{R}^3)}$ may be a polygon, namely: hexagon, pentagon, square or triangle. In these cases, the super- plane represented by (7) is called as hexagonal-, pentagonal-, quadragonal- and triangonal super–plane, respectively. For example, if we have the Euclidean plane represented by the equation

$$x + y - z = 0 \quad ; \quad x, y, z \in \mathbb{R} \tag{8}$$

the joint part is a (regular) hexagon

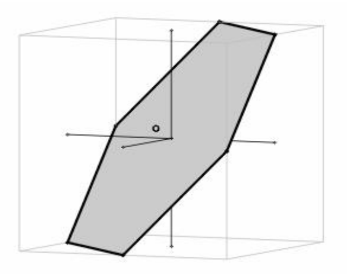

Fig. 1

Definition 2. Let us denote $\mathcal{P}_0 = \widetilde{P_0}$ and assume that $\mathcal{P}_0 \in \mathbb{R}^3$. The set $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}} = \widetilde{\mathbb{S}_{P_0;N}} \cap \mathbb{R}^3$, where $\mathcal{N} = \widetilde{N}$ (see the vectorial product after (1)) depending on $\mathbb{S}_{P_0;N} \cap \overline{(\mathbb{R}^3)}$, is called as hexagonal-, pentagonal-, quadragonal-, triangonal-**extra–plane**. The super-plane $\widetilde{\mathbb{S}_{P_0;N}}$ is named as the **holder** of extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ ■

Clearly the extra – plane is the visible part (in our universe) of its holder. Moreover, the border of extra – plane is already invisible in our universe. (See the Fig. 1. The border of haxagon is outside of $\underline{\mathbb{R}}^3$.) By (3) and Definition 1 we have

$$\mathcal{S}_{\mathcal{P}_0,\mathcal{N}} = \Big\{\mathcal{P} = (u, v, w) \in \mathbb{R}^3 \,|\, n_x \cdot \underline{u} + n_y \cdot \underline{v} + n_z \cdot \underline{w} = n_x \cdot \underline{u_0} + n_y \cdot \underline{v_0} + n_z \cdot \underline{w_0} \Big\} \tag{9}$$

where $\mathcal{P}_0 = (u_0, v_0, w_0) \in \mathbb{R}^3$ is a given point and $\mathcal{N} = (\widetilde{n_x}, \widetilde{n_y}, \widetilde{n_z}) \in \widetilde{\mathbb{R}^3}$ such that $\sqrt{n_x^2 + n_y^2 + n_z^2} = 1$ As u, v *and* w are real numbers, using (2) in [2] with $\sigma = 1$ (see [2], Part I.) by (9) we can see, that the extra plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is represented by the equation

$$n_x \cdot \tanh u + n_y \cdot \tanh v + n_z \cdot \tanh w = d \tag{10}$$

where $(n_x, n_y, n_y) \in \mathbb{R}^3$ such that $\sqrt{n_x^2 + n_y^2 + n_z^2} = 1$ and $d = n_x \cdot \underline{u_0} + n_y \cdot \underline{v_0} + n_z \cdot \underline{w_0}$ is a real number.

Example 1**.** Considering the equation (10) we may choose $\mathcal{P}_0 = \mathcal{O} = (0,0,0)$ and $n_x = n_y = \frac{1}{\sqrt{3}}$, $n_z = -\frac{1}{\sqrt{3}}$. This means that $\mathcal{N} = \left(\widetilde{\left(\frac{1}{\sqrt{3}}\right)}, \widetilde{\left(\frac{1}{\sqrt{3}}\right)}, \widetilde{\left(-\frac{1}{\sqrt{3}}\right)}\right)$ and (10) yields $\tanh u + \tanh v - \tanh w = 0$, $(u, v, w) \in \mathbb{R}^3$.

Hence, the extra–plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is represented by the equation $w = \tanh^{-1}(\tanh u + \tanh v)$; $-1 < \tanh u + \tanh v < 1$and has the next graph which shows a hexagonal extra – plane

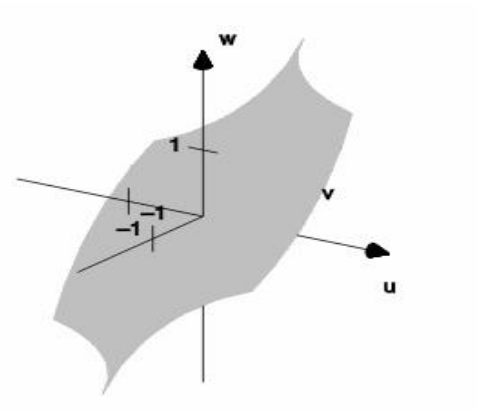

Fig. 2

Of course, the border-curve of this hexagonal extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is invisible in $\mathbb{R}^3$. On the other hand we are able to show it by piece by piece. For example, in the „depth" $w = \widetilde{(-1)}$ and the „height" $w = \breve{1}$ the holder of $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ description (5) gives the equations

$$v = -\tanh^{-1}(1 + \tanh u) \quad ; u < 0$$

and

$$v = \tanh^{-1}(1 - \tanh u) \quad ; u > 0,$$

representing „level – curves". Their projections are seen in the „u,v" coordinate – plane of $\mathbb{R}^3$:

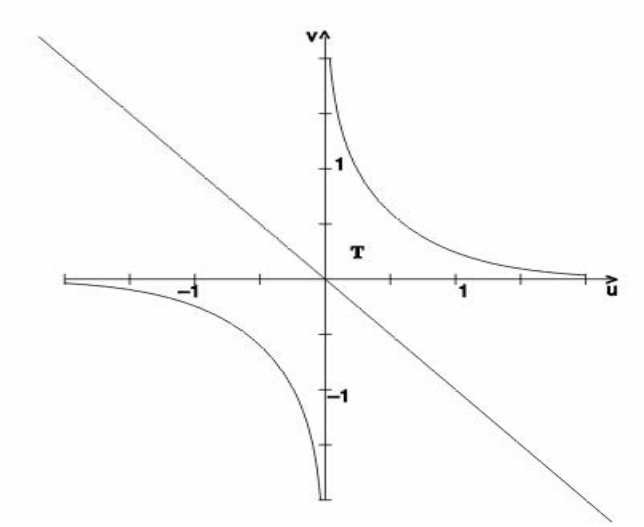

Fig. 3

Example 2. Let us consider the Euclideans plane described by the equation.

$$4x + 4y - 7z + 1 = 0.$$

(Considering (2) - (44) we may choose $P_0 = \left(0,0,\frac{1}{7}\right)$ and $N = \left(\frac{4}{9}, \frac{4}{9}, -\frac{7}{9}\right)$.) By (10) , the equation of the extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is

$$\frac{4}{7}\tanh u+\frac{4}{7}\tanh v-\tanh w=-\frac{1}{7},\quad (u,v,w)\in\mathbb{R}^3,$$

where $\mathcal{P}_0=\left(0,0,\widetilde{\left(\frac{1}{7}\right)}\right)$ and $\mathcal{N}=\left(\widetilde{\left(\frac{4}{9}\right)},\widetilde{\left(\frac{4}{9}\right)},\widetilde{\left(-\frac{7}{9}\right)}\right)$. Hence,

$$w=\tanh^{-1}\left(\frac{4}{7}\tanh u+\frac{4}{7}\tanh v+\frac{1}{7}\right)\quad ;\quad (u.v)\in\mathbb{R}^2 and\ \tanh u+\tanh v<\frac{3}{2}$$

The joint part of the Euclidean plane and $\overline{(\mathbb{R}^3)}$ is a pentagon having the peak – points

$(-1,-1,-1)$, $\left(1,-1,\frac{1}{7}\right)$, $\left(1,\frac{1}{2},1\right)$, $\left(\frac{1}{2},1,1\right)$ and $\left(-1,1,\frac{1}{7}\right)$,

so the extra- plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is a pentagonal extra - plane

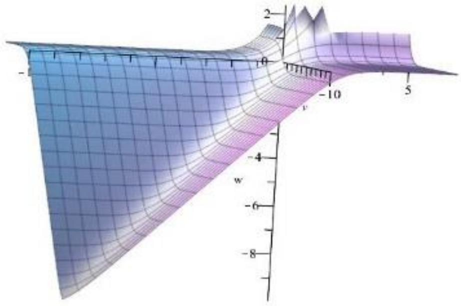

Fig. 4

The Fig. 4 makes perceptible the invisble border of $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is a super – pentagon with the peak-points $\left(\widetilde{(-1)},\widetilde{(-1)},\widetilde{(-1)}\right)$, $\left(\check{1},\widetilde{(-1)},\widetilde{\left(\frac{1}{7}\right)}\right)$, $\left(\check{1},\widetilde{\left(\frac{1}{2}\right)},\check{1}\right)$, $\left(\widetilde{\left(\frac{1}{2}\right)},\check{1},\check{1}\right)$ and $\left(\widetilde{(-1)},\check{1},\widetilde{\left(\frac{1}{7}\right)}\right)$.

Looking for the „level – curve" of $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ in the „depth" $w=\widetilde{(-1)}$ by (47) we can prove that necessarily $\underline{u}+\underline{v}=-2$. Considering that the point $(\underline{u},\underline{v})\in\overline{(\mathbb{R}^2)}$ (closed square) the solution $\underline{u}=-1\ and\ \underline{v}=-1$, only. This proves, that the „level – curve" is the point $\left(\widetilde{(-1)},\widetilde{(-1)},\widetilde{(-1)}\right)$, merely.

Example 3. Let us consider the extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ with $\mathcal{P}_0=(1.1,0)$ and $\mathcal{N}=\left(\widetilde{\left(\frac{1}{\sqrt{2}}\right)},\widetilde{\left(\frac{1}{\sqrt{2}}\right)},0\right)$. Now, (9) and (10) yield the equation

$$\tanh u+\tanh v=2\tanh 1\ \Leftrightarrow u\overline{\oplus}v=\check{2}\overline{\odot}1\quad ;(u,v)\in\mathbb{R}^2$$

Hence,

$$v=\tanh^{-1}(2\tanh 1-\tanh u)\quad ;\ u>\tanh^{-1}(2\tanh 1-1)\approx 0.6$$

so, the extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ has the graph

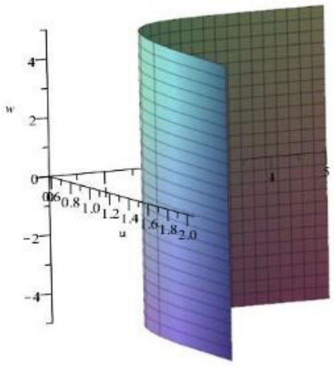

Fig. 5

The Fig, 5 makes perceptible the invisible border of $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is a super – rectangle with the peak-points

$$\left(\tanh^{-1}(2\tanh 1-1),\check{1},\widetilde{(-1)}\right),\left(\check{1},\tanh^{-1}(2\tanh 1-1),\widetilde{(-1)}\right),\left(\check{1},\tanh^{-1}(2\tanh 1-1),\check{1}\right)\text{ and }\left(\tanh^{-1}(2\tanh 1-1),\check{1},\check{1}\right)$$

So, $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is an extra – quadragonal extra – plane.

Example 4. Let us consider the extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ (see (9)) with $\mathcal{P}_0=\left(\widetilde{\left(\frac{2}{3}\right)}.\widetilde{\left(\frac{2}{3}\right)},\widetilde{\left(\frac{2}{3}\right)}\right)$ and $\mathcal{N}=\left(\widetilde{\left(\frac{1}{\sqrt{3}}\right)},\widetilde{\left(\frac{1}{\sqrt{3}}\right)},\widetilde{\left(\frac{1}{\sqrt{3}}\right)}\right)$. Now, (48) has the form

$$\tanh u+\tanh v+\tanh w=2\ ;\ (u,v,w)\in\mathbb{R}^3.$$

Hence, $w=\tanh^{-1}(2-\tanh u-\tanh v)$; $(u,v)\in\mathbb{R}^2\ \ and\ \ 1<\tanh u+\tanh v$.

Moreover, By (5) we can see that the super – plane containing the extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ has the equation

$$\frac{1}{\sqrt{3}}\cdot\underline{u}+\frac{1}{\sqrt{3}}\cdot\underline{v}+\frac{1}{\sqrt{3}}\cdot\underline{v}=\frac{2}{\sqrt{3}}\quad ;\quad (u,v,w)\in\widetilde{\mathbb{R}^3}$$

Hence, the border of the extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is a super – triangle, determined by the peak points

$$(\check{1},\check{1},0)\ ;\ (\check{1},0,\check{1})\ ;\ (0,\check{1},\check{1}).$$

So, the extra – plane $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ is a triangular extra – plane, perceived by the following Fig. 6.

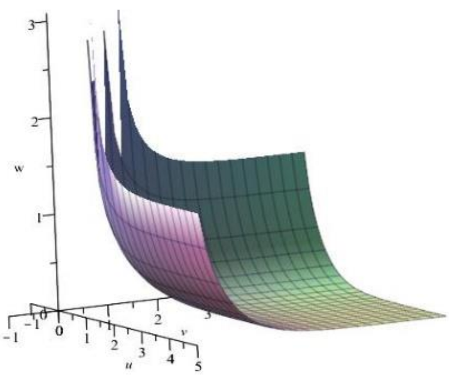

Fig. 6

III. Consequences of the Euclidean Geometry of the Multiverse on the Properties of Extra-Planes

If three or more points fall into an extra-plane (or super-plane) they are called extra- (or super-) complanar points. Clearly, extra-complanar points are super-complanar points, too. The Eucledian geometry of Multiverse $\breve{\mathbb{R}}^3$ partly covered in [1], Chapter 6, Sectiones 6.4 and 6.5. Here we have put together a more complete

Collection .1.

- **a)** For any two points, the two points are super – collinear. (See [1], Theorem 6.4.15.)
- **b)** If two super – lines contain the same two distinct points, then the two super – lines are equal.
- **c)** Every super – lines contain at least two distinct points.
- **d)** For any three points, the three points are super – coplanar. (See [1], Theorem 6.5.16.)
- **e)** If two super – planes each contain the same three non-extra-collinear points, then the two super- planes are equal.
- **f)** Every super-planes are non-empty.
- **g)** The non-empty junction of two super – planes contain at least two distinct points.
- **h)** If the super – plane contains two distinct points, then it includes the super – line that contains them.
- **i)** Multiverse contains four non super – coplanar points.

Theorem 1. Let be $\mathcal{P}_1 = (u_1, v_1, w_1), \mathcal{P}_2 = (u_2, v_2, w_2)$ *and* $, \mathcal{P}_3 = (u_3, v_3, w_3)$ distinc points of Multiverse such that they are non super – collinear and if

(i) all of them are in the universe $\mathbb{R}^3$

or

(ii) one of them is in the universe $\mathbb{R}^3$ and the another two points are situated on its border,

or

(iii) two of them are in the universe $\mathbb{R}^3$ and the third point is situated on the border of $\mathbb{R}^3$

or

(iv) all of them are situated on the border of the universe $\mathbb{R}^3$ such that these points do not situated on the same super – plane of the border,

then the box – phenomenon of the of the super - plane determined by the equation

$$\begin{vmatrix} \underline{u} & \underline{v} & \underline{w} & 1 \\ \underline{u_1} & \underline{v_1} & \underline{w_1} & 1 \\ \underline{u_2} & \underline{v_2} & \underline{w_2} & 1 \\ \underline{u_3} & \underline{v_3} & \underline{w_3} & 1 \end{vmatrix} = 0 \quad , (u, v, w) \in \breve{\mathbb{R}}^3, \tag{11}$$

gives an unambiguously determined extra – plane such that the super – plane contains the points $\mathcal{P}_1, \mathcal{P}_2$ *and* $\mathcal{P}_3$.

Proof. First, we remark that $\underline{\mathcal{P}_1}$, $\underline{\mathcal{P}_2}$ and $\underline{\mathcal{P}_3}$ are situated in $\mathbb{R}^3$ and they are non – collinear. (In the opposite case for the vectors $E_1 = \frac{\underline{\mathcal{P}_2} - \underline{\mathcal{P}_1}}{\|\underline{\mathcal{P}_2} - \underline{\mathcal{P}_1}\|}$ *and* $E_2 = \frac{\underline{\mathcal{P}_3} - \underline{\mathcal{P}_1}}{\|\underline{\mathcal{P}_3} - \underline{\mathcal{P}_1}\|}$ we have that $E_1 = E_2$ or $E_1 = -E_2$ and by (12) - (14) (seen in [2]) we get that the points $\mathcal{P}_1, \mathcal{P}_2$ *and* $\mathcal{P}_3$.are super – collinear.)

Second, we consider the Euclidean plane

$$\begin{vmatrix} x & y & z & 1 \\ \underline{u_1} & \underline{v_1} & \underline{w_1} & 1 \\ \underline{u_2} & \underline{v_2} & \underline{w_2} & 1 \\ \underline{u_3} & \underline{v_3} & \underline{w_3} & 1 \end{vmatrix} = 0 \quad , (x, y, z) \in \mathbb{R}^3, \tag{12}$$

which by Axioms of Euclidean geometry is unambiguously determined and contains the points $\underline{\mathcal{P}_1}$, $\underline{\mathcal{P}_2}$ and $\underline{\mathcal{P}_3}$.

Third, we use that tha mapping between the points (x, y, z) *and* $(\breve{x}, \breve{y}, \breve{z})$ is simultaneosly unambiguous so, using the nominations $u = \breve{x}, v = \breve{y}$ and $w = \breve{z}$ by (12) we have that the equation (11) determines a super – plane which contains the points $\mathcal{P}_1, \mathcal{P}_2$ *and* $\mathcal{P}_3$.

Finally, we observes that in the the cases (i) – (iv) this super – plane and the our universe has a joint part, so, the points $(u, v, w) \in \mathbb{R}^3$ satisfying the equation (12) form an extra – plane. ■

Remark 1. The reduced (11) is the best description of extra – planes by the equation

$$\begin{vmatrix} \tanh u & \tanh v & \tanh w & 1 \\ \underline{u_1} & \underline{v_1} & \underline{w_1} & 1 \\ \underline{u_2} & \underline{v_2} & \underline{w_2} & 1 \\ \underline{u_3} & \underline{v_3} & \underline{w_3} & 1 \end{vmatrix} = 0 \quad , (u, v, w) \in \mathbb{R}^3,$$

(13)

where $\mathcal{P}_1 = (u_1, v_1, w_1), \mathcal{P}_2 = (u_2, v_2, w_2)$ *and* $, \mathcal{P}_3 = (u_3, v_3, w_3)$ are situated on the border of the universe $\mathbb{R}^3$ such that these points do not situated on the same super – plane of the border. (See Theorem 1, the case (iv).)

Exercise 1. Let us give the equation of extra–plane having the peak-points $\mathcal{P}_1 = \left(\breve{(-1)}, \breve{(-1)}, \breve{(-1)}\right), \mathcal{P}_2 = \left(\breve{1}, \breve{(-1)}, \breve{\left(\frac{1}{7}\right)}\right), \mathcal{P}_3 = \left(\breve{(-1)}, \breve{1}, \breve{\left(\frac{1}{7}\right)}\right)$. Has it other peak–points, yet?

Solution. Fist we ascertain that the points $\mathcal{P}_1, \mathcal{P}_2$ *and* $\mathcal{P}_3$ are situated on the border of the universe $\mathbb{R}^3$ but they are not on same super – plane of the border. (The points $\mathcal{P}_1 = \left(\breve{(-1)}, \breve{(-1)}, \breve{(-1)}\right)$ *and* $\mathcal{P}_2 = \left(\breve{1}, \breve{(-1)}, \breve{\left(\frac{1}{7}\right)}\right)$ are on the super – plane characterized by $v = \breve{(-1)}$ but the point $\mathcal{P}_3$ is not there.) Second, we use the equation (51)

$$\begin{vmatrix} \tanh u & \tanh v & \tanh w & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & 1/7 & 1 \\ -1 & 1 & 1/7 & 1 \end{vmatrix} = 0$$

Using the Sarrus – rule

$$\begin{vmatrix} \tanh u & \tanh v & \tanh w & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & 1/7 & 1 \\ -1 & 1 & 1/7 & 1 \end{vmatrix} = (\tanh u) \cdot \left(-\frac{16}{7}\right) - (\tanh v) \cdot \left(\frac{16}{7}\right) + (\tanh w) \cdot 4 - \frac{4}{7}$$

equation $4 \tanh u + 4 \tanh v - 7 \tanh w + 1 = 0$ is obtained. Returning to the Example 3 our extra – plane is just found there.

We may believe, that by the given points $\mathcal{P}_1, \mathcal{P}_2$ *and* $\mathcal{P}_3$ the requested extra – plane seems to be a triangular extra – plane, but the Example 2 says that it is pentagonal. Considering (11) we have the equation of the super–plane containing the requested extra plane

$$\begin{vmatrix} \underline{u} & \underline{v} & \underline{w} & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & 1/7 & 1 \\ -1 & 1 & 1/7 & 1 \end{vmatrix} = 0 \quad , (u, v, w) \in \breve{\mathbb{R}}^3$$

Having points $\mathcal{P}_4 = \left(\breve{1}, \breve{\left(\frac{1}{2}\right)}, \breve{1}\right)$ and $\mathcal{P}_5 = \left(\breve{\left(\frac{1}{2}\right)}, \breve{1}, \breve{1}\right)$ we can controll that the latter equation fulfils, so the requested extra – plane has five peak–points. ■Istennek Hála, 2017.08.02. 5:17.SzI

Theorem 1 and Collection 1 with Property 3 (see in [2], (29)) yield

Corollary 1. Let be $\mathcal{P}_0 = (u_0, v_0, w_0) \in \mathbb{R}^3, \mathcal{B}_1 = (u_1, v_1, w_1)$ *and* $, \mathcal{B}_2 = (u_2, v_2, w_3)$ be distinc and non super- collinear points of the Multiverse. Moreover, let be the points $\mathcal{B}_1$ *and* $\mathcal{B}_2$ on the border of our universe such that they are not situated on the same super – plane of border. Then these three points determine an extra – plane such that the extra–plane contains the point $\mathcal{P}_o$ and the extra–line $\mathbb{L}^{extra}_{(\mathcal{B}_1,\mathcal{B}_2)}$. (In short $\mathbb{L}^{extra}_{(\mathcal{B}_1,\mathcal{B}_2)} = \mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[.)$.

Exercise 2. Let us give the equation of extra–plane having the peak-points $\mathcal{B}_1 = (\widetilde{(-1)}, \breve{1}, 0)$ $, \mathcal{B}_2 = (\breve{1}, \widetilde{(-1)}, 0)$ and the point $\mathcal{P}_0$= $\left(\widetilde{\left(\frac{1}{4}\right)}, \widetilde{\left(\frac{1}{4}\right)}, \widetilde{\left(\frac{1}{2}\right)}\right)$ of our universe. Let us prove that this extra-plane contains the extra–line $\mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[)$.

Solution. Fist we ascertain that the points $\mathcal{B}_1$ *and* $\mathcal{B}_2$ are situated on the border of the universe $\mathbb{R}^3$ but they are not on same super – plane of the border. Moreover, $\mathcal{P}_0 \in \mathbb{R}^3$ and the points $\mathcal{P}_0, \mathcal{B}_1$ *and* $\mathcal{B}_2$ are are non super– collinear. (The points $(-1,1,0); (1.-1,0)$ *and* $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$ are non collinear.)

Second, we use the equation (13)

$$\begin{vmatrix} \tanh u & \tanh v & \tanh w & 1 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 \\ 1 & -1 & 0 & 1 \\ -1 & 1 & 0 & 1 \end{vmatrix} = 0\,, \quad (u, v, w) \in \mathbb{R}^3.$$

As

$$\begin{vmatrix} \tanh u & \tanh v & \tanh w & 1 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 \\ 1 & -1 & 0 & 1 \\ -1 & 1 & 0 & 1 \end{vmatrix} = \frac{1}{4} \cdot \begin{vmatrix} \tanh u & \tanh v & \tanh w & 1 \\ 1 & 1 & 2 & 4 \\ 1 & -1 & 0 & 1 \\ -1 & 1 & 0 & 1 \end{vmatrix}$$
$$= \frac{1}{4} \cdot \begin{vmatrix} \tanh u & \tanh v & \tanh w & 1 \\ 1 & 1 & 2 & 4 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{vmatrix} =$$
$$= \frac{1}{2} \cdot \begin{vmatrix} \tanh u & \tanh v & \tanh w \\ 1 & 1 & 2 \\ 1 & -1 & 0 \end{vmatrix}$$
$$= \tanh u + \tanh v - \tanh w,$$

so. the equation of requested extra plane has the equation $\tanh u + \tanh v - \tanh w = 0, (u, v, w) \in \mathbb{R}^3$. By Example 1 we can see it on Fig.2. This extra – plane contains $\mathcal{P}_0$ because $\tanh \widetilde{\left(\frac{1}{2}\right)} + \tanh \widetilde{\left(\frac{1}{2}\right)} - \tanh \widetilde{\left(\frac{1}{4}\right)} = 0$.

Using Theorem 1.10 (found in [3]) we have that the extra–plane $\mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[)$ is desribed by the equation system

$$\begin{cases} u = \tau \overline{\odot} \widetilde{\left(-\frac{1}{\sqrt{2}}\right)} \\ v = \tau \overline{\odot} \widetilde{\left(\frac{1}{\sqrt{2}}\right)} \\ w = 0 \end{cases} , \quad \widetilde{(-1)} < \tau < \breve{1}.$$

So, the extra – line has the equation $v = -u$ and $w = 0$, and it is an Euclidean line, too. Clearly, the equation $\tanh u + \tanh v - \tanh w = 0$ fulfiils, so, the the extra – plane contains the the extra–line $\mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[)$.

Exercise 3. Let us find the polygonial - type of extra – plane having the point $\mathcal{P}_0 = (1,1,1)$ and peak – points $\mathcal{B}_1 = \left(2, \tanh^{-1}(2\tanh 1 - \tanh 2), \breve{1}\right)$ *and* $\mathcal{B}_2 = \left(\tanh^{-1}(2\tanh 1 - \tanh 2), 2, \widetilde{(-1)}\right)$.

Solution. Fist we observe that the points $\mathcal{P}_0, \mathcal{B}_1$ *and* $\mathcal{B}_2$ are are non super – collinear. (Because, the points $(\tanh 2, 2\tanh 1 - \tanh 2, 1); (2\tanh 1 - \tanh 2, \tanh 2, -1)$ and $(\tanh 1, \tanh 1, \tanh 1)$ are non collinear.) Second, we use the equation (11)

$$\begin{vmatrix} \underline{u} & \underline{v} & \underline{w} & 1 \\ \tanh 1 & \tanh 1 & \tanh 1 & 1 \\ \tanh 2 & 2\tanh 1 - \tanh 2 & 1 & 1 \\ 2\tanh 1 - \tanh 2 & \tanh 2 & -1 & 1 \end{vmatrix} = 0\,, \quad (u, v, w) \in \mathbb{R}^3.$$

is obtained which is the equation of the super – plane containing the investigated extra–plane.

Computing determinant

$$\begin{vmatrix} \underline{u} & \underline{v} & \underline{w} & 1 \\ \tanh 1 & \tanh 1 & \tanh 1 & 1 \\ \tanh 2 & 2\tanh 1 - \tanh 2 & 1 & 1 \\ 2\tanh 1 - \tanh 2 & \tanh 2 & -1 & 1 \end{vmatrix} = \underline{u} \cdot$$
$$\left(2(\tanh 1)(\tanh 2)\right) - 2(\tanh 1)^2 - \underline{v} \cdot \left(2(\tanh 1)^2 - 2(\tanh 1)(\tanh 2)\right) + \underline{w} \cdot 0 - 1 \cdot (4(\tanh 1)^2(\tanh 2) - 4(\tanh 1)^3)$$

we have that the super-plane (containing the investigated extra–plane) has the equation

$$\underline{u} + \underline{v} = 2\tanh 1\,, \qquad (u.v.w) \in \widetilde{\mathbb{R}^3} \tag{14}$$

Our universe is an open „big cube"of the Multiverse bordered by six super–planes described by the equations

$$\text{„Left" border: } \underline{u} = -1 \quad , \ (u.v.w) \in \widetilde{\mathbb{R}^3} \tag{15}$$

$$\text{„Right" border: } \underline{u} = 1 \quad , \ (u.v.w) \in \widetilde{\mathbb{R}^3} \tag{16}$$

$$\text{„Before" border: } \underline{v} = -1 \quad , \ (u.v.w) \in \widetilde{\mathbb{R}^3} \tag{17}$$

$$\text{„Back" border: } \underline{v} = 1 \quad , \ (u.v.w) \in \widetilde{\mathbb{R}^3} \tag{18}$$

$$\text{„Below" border: } \underline{w} = -1 \quad , \ (u.v.w) \in \widetilde{\mathbb{R}^3} \tag{19}$$

And

$$\text{„Upper" border: } \underline{w} = 1 \quad , \ (u.v.w) \in \widetilde{\mathbb{R}^3} \tag{20}$$

respectively. The universe $\mathbb{R}^3$ together its border forms the „closed universe"

$$\overline{\mathbb{R}^3} = \left\{ \mathcal{P} = (u, v, w) \in \widetilde{\mathbb{R}^3} \middle| \begin{cases} \widetilde{(-1)} \le u \le \breve{1} \\ \widetilde{(-1)} \le v \le \breve{1} \\ \widetilde{(-1)} \le w \le \breve{1} \end{cases} \right\} \tag{21}$$

The equation system (14)-(15) yields $\underline{v} = 2\tanh 1 + 1 > 1$. By (21) we have that $|\underline{v}| \le 1$, so in the „left" border there is no point of the border of the investigated extra–plane. The equation system (14)-(16) yields that $\underline{v} = 2\tanh 1 - 1$, and considering (21) we get the endpoints

$\mathcal{B}_{below-right} = \left(\breve{1}, \tan^{-1}(2\tanh 1 - 1), \widetilde{-1}\right)$ and $\mathcal{B}_{upper-right} = \left(\breve{1}, \tan^{-1}(2\tanh 1 - 1), \breve{1}\right) \in \overline{\mathbb{R}^3}$,so the super – passage

$$\mathcal{L}([\mathcal{B}_{below-right}, \mathcal{B}_{upper-right}]) = \left\{\mathcal{P} = (u, v, w) \in \widetilde{\mathbb{R}^3} \,\middle|\, \left\{\begin{matrix} u = \breve{1} \\ v = \tan^{-1}(2\tanh 1 - 1) \\ \widetilde{(-1)} \le w \le \breve{1} \end{matrix}\right\}\right.$$

is a part of the border of the investigated extra – plane. The equation system (14)-(17) yields $\underline{u} = 2\tanh 1 + 1 > 1$. By (21) we have that $|\underline{u}| \le 1$, so in the „before" border there is no point of the border of the investigated extra – plane. Similarly, the equation system (14)-(18) yields $\underline{u} = 2\tanh 1 - 1$ so, the super – passage

$$\mathcal{L}([\mathcal{B}_{below-back}, \mathcal{B}_{upper-back}]) = \left\{\mathcal{P} = (u, v, w) \in \widetilde{\mathbb{R}^3} \,\middle|\, \left\{\begin{matrix} u = \tan^{-1}(2\tanh 1 - 1) \\ v = \breve{1} \\ \widetilde{(-1)} \le w \le \breve{1} \end{matrix}\right\}\right.$$

is a newer part of the border of the investigated extra – plane.

Considering the endpoints

$$\mathcal{B}_{below-back} = \left(\tan^{-1}(2\tanh 1 - 1, \breve{1}, \widetilde{(-1)})\right) \in \overline{\mathbb{R}^3},$$
$$\mathcal{B}_{below-right} = \left(\breve{1}, \tan^{-1}(2\tanh 1 - 1), \widetilde{(-1)}\right) \in \overline{\mathbb{R}^3}$$

by the equation system (14)-(19) gives that $\mathcal{L}([\mathcal{B}_{below-back}, \mathcal{B}_{below-right}])$ borderes the investigated extra – plane. Finally, the points

$$\mathcal{B}_{upper-back} = \left(\tan^{-1}(2\tanh 1 - 1, \breve{1}, \breve{1})\right) \in \overline{\mathbb{R}^3}$$

And

$$\mathcal{B}_{upper-right} = \left(\breve{1}, \tan^{-1}(2\tanh 1 - 1), \breve{1}\right) \in \overline{\mathbb{R}^3}$$

by the equation system (14)-(20) gives the last part of the border of of the investigated extra – plane. So, we have that the the investigated extra – plane is a quadragonal extra – plane. (We can see it on Fig. 5.)

Observation 1. By Example 2 we can see that the point $\mathcal{P}_0 = (1.1,0) \in \mathbb{R}^3$ and the super-vector $\mathcal{N} = \left(\widetilde{\left(\frac{1}{\sqrt{2}}\right)}, \widetilde{\left(\frac{1}{\sqrt{2}}\right)}, 0\right)$ determines the extra – plane

$$\mathcal{S}_{\mathcal{P}_0,\mathcal{N}} = \{(u, v, w) \in \mathbb{R}^3 | \breve{u} \widetilde{\oplus} \breve{v} = \breve{2} \widetilde{\odot} 1 \; ; \; (u, v) \in \mathbb{R}^2 \; and \; w \in \mathbb{R}\}.$$

(See Fig.5.) It is easy to see that the point $\mathcal{P} = (1,1,1)$ is a point of $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$ and the extra – line

$$\mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[) = \left\{\begin{matrix} u = 1 \widetilde{\oplus} \left(\tau \widetilde{\odot} \left(\frac{\widetilde{\tanh 1 - \tanh 2}}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}\right)\right) \\ v = 1 \widetilde{\oplus} \left(\tau \widetilde{\odot} \left(\frac{\widetilde{\tanh 2 - \tanh 1}}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}\right)\right) \\ w = \tau \widetilde{\odot} \left(-\frac{\widetilde{1}}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}\right) \end{matrix}\right. ,$$
$$\left(-\widetilde{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}\right) < \tau < \left(\widetilde{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}\right)$$

determined by the border – points $\mathcal{B}_1 = \left((2\tanh \widetilde{1 - \tanh 2}), 2, \widetilde{(-1)}\right)$ and $\mathcal{B}_2 = \left(2, (2\tanh \widetilde{1 - \tanh 2}), \breve{1}\right)$ is situated on $\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}$.

Conversely, we will verify that the point $\boldsymbol{\mathcal{P}} = (\mathbf{1}, \mathbf{1}, \mathbf{1}) \notin \boldsymbol{\mathcal{L}}(]\boldsymbol{\mathcal{B}_1}, \boldsymbol{\mathcal{B}_2}[)$ together with the extra – line $\boldsymbol{\mathcal{L}}(]\boldsymbol{\mathcal{B}_1}, \boldsymbol{\mathcal{B}_2}[)$ determine the extra – plane $\boldsymbol{\mathcal{S}_{\mathcal{P}_0,\mathcal{N}}}$.

First verification. In Exercise 3 we can see that the super - plane containing the points $\mathcal{P}, \mathcal{B}_1$ *and* $\mathcal{B}_2$ had the equation (14). Hence, the extra – plane has the equation

$$\underline{u} + \underline{v} = 2\tanh 1, \qquad (u.v.w) \in \mathbb{R}^3.$$

Exploding both sides, we have the equation $\breve{u} \widetilde{\oplus} \breve{v} = \breve{2} \widetilde{\odot} 1$.

Second verification. As $\mathcal{P}_0 \in \mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[)$ it is sufficient to show, that the point $\mathcal{P} = (1,1,1)$ and the extra – line $\mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[)$ determine $\mathcal{N}$. To reach this aim, we consider the (Euclidean) plane determined by the point $\underline{\mathcal{P}} = (\tanh 1, \tanh 1, \tanh 1)$ and the (Euclidean) passage

$$\underline{\mathcal{L}(]\mathcal{B}_1, \mathcal{B}_2[)} = \left\{\begin{matrix} \underline{u} = \tanh 1 + \underline{\tau} \cdot \frac{\tanh 1 - \tanh 2}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}} \\ \underline{v} = \tanh 1 + \underline{\tau} \cdot \frac{\tanh 2 - \tanh 1}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}} \\ \underline{w} = -\underline{\tau} \cdot \frac{1}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}} \end{matrix}\right. , -\sqrt{2(\tanh 1 - \tanh 2)^2 + 1} < \underline{\tau} < \sqrt{2(\tanh 1 - \tanh 2)^2 + 1}.$$

Clearly, the line determined by the points $\underline{\mathcal{P}_0} = (\tanh 1, \tanh 1, 0)$ and $\underline{\mathcal{P}} = (\tanh 1, \tanh 1, \tanh 1)$ is also situated on the (Euclidean) plane and its direction vector is (0,0,1). Considering the vectorial product of

$$E_1 = \left(\frac{\tanh 1 - \tanh 2}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}, \frac{\tanh 2 - \tanh 1}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}, -\frac{1}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}\right)$$

and E_2=(0,0,1) is

$$E_1 \times E_2 = \left(\frac{\tanh 2 - \tanh 1}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}, \frac{\tanh 2 - \tanh 1}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}, 0\right).$$

Hence, $\underline{\mathcal{N}} = \frac{1}{\|E_1 \times E_2\|} \cdot (E_1 \times E_2)$. Moreover, $\|E_1 \times E_2\| = \sin \measuredangle(E_1, E_2) > 0$.

On the other hand by inner product $E_1 \cdot E_2 = -\frac{1}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}} = \cos \measuredangle(E_1, E_2)$. As

$$\sin \measuredangle(E_1, E_2) = \sqrt{1 - \frac{1}{2(\tanh 1 - \tanh 2)^2 + 1}} = \frac{\sqrt{2} \cdot (\tanh 2 - \tanh 1)}{\sqrt{2(\tanh 1 - \tanh 2)^2 + 1}}.$$

So,

$$\underline{\mathcal{N}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

and finally we get

$$\mathcal{N} = \left(\widetilde{\left(\frac{1}{\sqrt{2}}\right)}, \widetilde{\left(\frac{1}{\sqrt{2}}\right)}, 0\right).$$ *Istennek Hála, 2017. aug. 14 – 16.58. Sz. I.*

Using the notation $]|\mathcal{V}|[= \widetilde{(\|\mathcal{V}\|)}$, $\mathcal{V} \in \widetilde{\mathbb{R}^3}$ we give

Theorem 2. Let be $\mathcal{P}_0 = (u_0, v_0, w_0) \in \mathbb{R}^3$, $\mathcal{B}_I = (u_I, v_I, w_I)$ *and* $\mathcal{B}_{II} = (u_{II}, v_{II}, w_{II}) \in (\overline{\mathbb{R}^3} \setminus \mathbb{R}^3)$ distinc points of Multiverse such that they are non super–collinear. Then the half extra-lines

$$\mathcal{L}([\mathcal{P}_0, \mathcal{B}_I[) = \left\{\mathcal{P} \in \mathbb{R}^3 | \mathcal{P} = \mathcal{P}_0 \overline{\oplus} \left(\tau \overline{\odot} \left((\mathcal{B}_I \overline{\ominus} \mathcal{P}_0) \overline{\oslash} \,]|\mathcal{B}_I \overline{\ominus} \mathcal{P}_0|[\right) \right) \right\}, \quad 0 \le \tau < \,]|B_I \overline{\ominus} \mathcal{P}_0|[\tag{22}$$

and

$$\mathcal{L}([\mathcal{P}_0, \mathcal{B}_{II}[) = \left\{\mathcal{P} \in \mathbb{R}^3 | \mathcal{P} = \mathcal{P}_0 \overline{\oplus} \left(\sigma \overline{\odot} \left((\mathcal{B}_I \overline{\ominus} \mathcal{P}_0) \overline{\oslash} \,]|\mathcal{B}_{II} \overline{\ominus} \mathcal{P}_0|[\right) \right) \right\}, 0 \le \sigma < \,]|B_{II} \overline{\ominus} \mathcal{P}_0|[\tag{23}$$

determines an extra – plane which contains the extra–lines determined by the point-pairs $(\mathcal{P}_0, \mathcal{B}_I)$ and $(\mathcal{P}_0, \mathcal{B}_{II})$, respectively.

Proof. It is easy to see that B_I is the border-point of the half extra–line $\mathcal{L}([\mathcal{P}_0, \mathcal{B}_I[)$. Similarly, B_{II} is the border-point of the half extra–line $\mathcal{L}([\mathcal{P}_0, \mathcal{B}_{II}[)$. By Collection 1,d) and the non super – collinear points P_0, B_I and B_{II} determine the super–plane containing them. Moreover, by Theorem 1(ii) this super-plane is the holder of the extra-plane, determined by P_0, B_I and B_{II}. So, the extra– lines determined by the point-pairs $(\mathcal{P}_0, \mathcal{B}_I)$ and the $(\mathcal{P}_0, \mathcal{B}_{II})$ are also situated on this extra-plane. (See Property 2 in [2].) On the other hand exploding the box – phenomenons of the super-plane and super–lines we get the stated extra–plane and extra–lines.■

Example 5. Let be given the point $\mathcal{P}_0 = \mathcal{O}$, $\mathcal{B}_I = \left(\widetilde{\left(\tfrac{1}{2}\right)}, \widetilde{\left(\tfrac{1}{2}\right)}, \breve{1}\right)$ *and* $\mathcal{B}_{II} = \left(\breve{1}, \widetilde{(-1)}, 0\right)$ Clearly, they are non super – collinear. By (22) and (23) the we consider the half extra-lines

$$\mathcal{L}([\mathcal{O}, \mathcal{B}_I[) = \left\{ (u, v, w) \in \mathbb{R}^3 \middle| \begin{matrix} u = \tanh^{-1}\frac{t}{\sqrt{6}} \\ v = \tanh^{-1}\frac{t}{\sqrt{6}} \\ w = \tanh^{-1}\frac{2t}{\sqrt{6}} \end{matrix} \right\}, \text{ where } 0 \le t < \frac{\sqrt{6}}{2} \tag{24}$$

And

$$\mathcal{L}([\mathcal{O}, \mathcal{B}_{II}[) = \left\{ (u, v, w) \in \mathbb{R}^3 \middle| \begin{matrix} u = \tanh^{-1}\frac{s}{\sqrt{2}} \\ v = -\tanh^{-1}\frac{s}{\sqrt{6}} \\ w = 0 \end{matrix} \right\}, \quad \text{where } 0 \le s < \sqrt{2} \tag{25}$$

By Theorem 1 (ii) the points $\mathcal{O}, B_I$ *and* B_{II} determine the extra–plane

$$\begin{vmatrix} \underline{u} & \underline{v} & \underline{w} & 1 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 1 & 1 \\ 1 & -1 & 0 & 1 \end{vmatrix} = \underline{u} + \underline{v} - \underline{w} = 0 \quad , (u, v, w) \in \mathbb{R}^3 \tag{26}$$

This extra – plane is seen on the Fig. 2.

By (62) the extra–line determined by the point-pair $(\mathcal{P}_0, \mathcal{B}_I)$ has the equation – system

$$\left\{ (u, v, w) \in \mathbb{R}^3 \middle| \begin{matrix} u = \tanh^{-1}\frac{t}{\sqrt{6}} \\ v = \tanh^{-1}\frac{t}{\sqrt{6}} \\ w = \tanh^{-1}\frac{2t}{\sqrt{6}} \end{matrix} \right\}, \text{ where } -\frac{\sqrt{6}}{2} < t < \frac{\sqrt{6}}{2} \tag{27}$$

and it is shown in the next figure

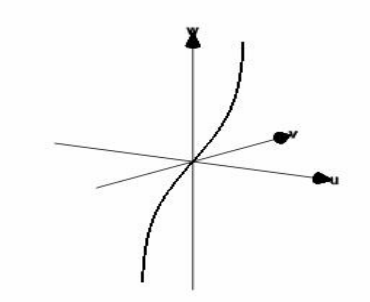

Fig. 7

moreover, we can see, that it is situated on the extra – plane given by (26)

By (25) the extra – line determined by the point-pair $(\mathcal{P}_0, \mathcal{B}_{II})$ has the equation – system

$$\left\{ (u, v, w) \in \mathbb{R}^3 \middle| \begin{matrix} u = \tanh^{-1}\frac{s}{\sqrt{2}} \\ v = -\tanh^{-1}\frac{s}{\sqrt{6}} \\ w = 0 \end{matrix} \right\}, \text{ where } -\sqrt{2} < s < \sqrt{2} \tag{28}$$

and it is shown on the Fig. 3, moreover, we can see, that it is situated on the extra–plane given by (26). This extra – plane is seen on the Fig. 2.

Example 6. Let be given the point $\mathcal{P}_1 = \left(\breve{1}, \widetilde{\left(\tfrac{1}{2}\right)}, 0\right)$, $\mathcal{P}_2 = \left(\widetilde{\left(\tfrac{1}{2}\right)}, \breve{1}, 0\right)$ $\mathcal{P}_3 = \left(\widetilde{\left(\tfrac{1}{2}\right)}, \widetilde{\left(\tfrac{1}{2}\right)}, \breve{1}\right)$ Clearly, they are non super – collinear. We consider the extra-lines given by their endpoints (see Property 3 in [2])

$$\mathcal{L}(]\mathcal{P}_1, \mathcal{P}_3[) = \left\{ (u, v, w) \in \mathbb{R}^3 \middle| \begin{matrix} u = \tanh^{-1}\left(1 - \frac{t}{2}\right) \\ v = \tanh^{-1}\frac{1}{2} \\ w = \tanh^{-1} t \end{matrix} \right\}, \text{ where } 0 < t < 1 \tag{29}$$

And

$$\mathcal{L}(]\mathcal{P}_2, \mathcal{P}_3[) = \left\{ (u, v, w) \in \mathbb{R}^3 \middle| \begin{matrix} u = \tanh^{-1}\frac{1}{2} \\ v = \tanh^{-1}\left(1 - \frac{s}{2}\right) \\ w = \tanh^{-1} s \end{matrix} \right\}, \text{ where } 0 < s < 1 \tag{30}$$

Having the joint endpoint $\mathcal{P}_3$ they are extra parallels of each other. (See Definition 1 in [2].)

By Theorem 1 (iv) the points $\mathcal{P}_1, \mathcal{P}_2$ *and* $\mathcal{P}_3$ determine the triangular extra–plane

$$\begin{vmatrix} \underline{u} & \underline{v} & \underline{w} & 1 \\ 1/2 & 1/2 & 1 & 1 \\ 1 & 1/2 & 0 & 1 \\ 1/2 & 1 & 0 & 1 \end{vmatrix} = 2\underline{u} + 2\underline{v} + \underline{w} - 3 = 0 \Leftrightarrow$$

$$(\breve{2} \overline{\odot} u) \overline{\oplus} (\breve{2} \overline{\odot} v) \overline{\oplus} w = \breve{3}, (u, v, w) \in \mathbb{R}^3. \text{ (See (11))} \tag{31}$$

Its graph having the equation

$w = \tanh^{-1}(3 - 2\tanh u - 2\tanh v)$, where $(u, v) \in \mathbb{R}^2$ and $\tanh u + \tanh v > 1$

is seen in the following figure

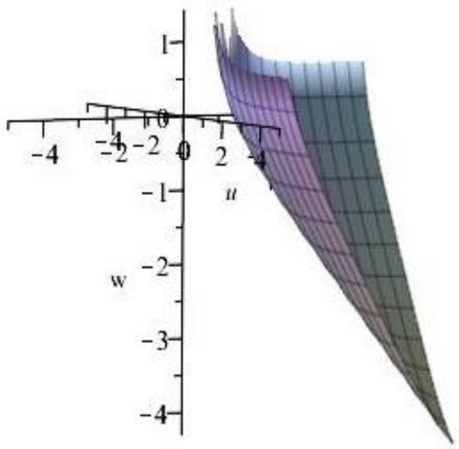

Fig 8.

The peak-points are $(\breve{1}, \breve{1}, \overline{-1}), (\breve{1}, 0, \breve{1})$ and $(0, \breve{1}, \breve{1})$

By (29)-(31) we can see, that the extra parallels extra-lines $\mathcal{L}(]\mathcal{P}_1, \mathcal{P}_3[)$ and $\mathcal{L}(]\mathcal{P}_2, \mathcal{P}_3[)$ are in the extra-plane having the equation

$$(\breve{2} \overline{\odot} u) \overline{\oplus} (\breve{2} \overline{\odot} v) \overline{\oplus} w = \breve{3} \quad , \quad (u, v, w) \in \mathbb{R}^3 \qquad (32)$$

*Observation 2***.** We can see that

- two extra lines cutting each other (see (24) and (25))
- two (distinc) extra parallel extra – lines (29) and (30)

determine an extra – plane.

IV. Extra Parallelism Relations between Extra-Lines and Extra-Planes

Definition 2. Assuming that the extra–line does not situate on the extra–plane but they have a joint border–point on $\overline{\mathbb{R}^3} \setminus \mathbb{R}^3$ then they are called extra parallel.

*Example 7***.** The extra-line given by the equation system (27) (see Fig. 7) and the (triangular) extra–plane represented by the equation (32) (see Fig.8) are extra parallel. (The joint border–point is $\left(\widetilde{\left(\frac{1}{2}\right)}, \widetilde{\left(\frac{1}{2}\right)}, \breve{1}\right)$. See Fig. 3.)

In the Euclidean geometry the relation between the lines and planes is relatively simple:

- They have two joint points. (The line is situated on the plane.)
- They have one joint points, only. (The line cuts through the plane.)
- They have no joint points. (They are parallel with each other.)

As it was already mentioned in the geometry of Multiverse the situations are similar to the Euclidean geometry of our universe. (See Collection.1.) But the extra-geometry of our universe is more complicated: If an extra–line and an extra – plane.

- **I.)** have two joint points then the extra–line is situated on the extra–plane,
- **II.)** have only one joint points, then the extra-line cuts through the extra-plane.)
- **III.)** have no joint points then we three further cases are possible:

  Case a) the extra–line is extra parallel with extra–plane (see Example 7),

  Case b) the holders of exra–line and of extra–plane have only one joint points outside $\overline{\mathbb{R}^3}$,

  Case c) the holders of exra–line and of extra–plane are super parallels. (Their holders have not any joint point in $\widetilde{\mathbb{R}^3}$.)

Definition 3. The extra – line and extra–plane,

- in the of Case b) are called first–type missing elements of the space $\mathbb{R}^3$
- in the of Case c) are called second – type missing elements of the space $\mathbb{R}^3$.

Example 8. The extra-line given by the equation system

$$\left\{(u, v, w) \in \mathbb{R}^3 \,\middle|\, \begin{matrix} u=0 \\ v=0 \\ w=\tau \end{matrix}\right\}, \ \widetilde{(-1)} < \tau < \breve{1}, \text{ (this is the „w" axis),}$$

and triangular extra–plane (see Fig. 8), given by (32) are first–type missing elements of our universe.

Really, the holder of extra – line is the super – line described by the equation system

$$\left\{(u, v, w) \in \mathbb{R}^3 \,\middle|\, \begin{matrix} u = 0 \\ v = 0 \\ w = \tau \end{matrix}\right\} \quad , \quad \tau \in \breve{\mathbb{R}}$$

and the holder of extra – plane is the super - plane given by the equation (32) have their joint point $(0,0,\breve{3}) \in \widetilde{\mathbb{R}^3}$,which is not seen on the the Fig. 8. But, we can catch its sight in an another universe of the Multiverse, by the super-shift transformation

$$\begin{cases} \xi = u \\ \eta = v \\ \zeta = w \overline{\ominus} \widetilde{\left(\frac{5}{2}\right)} \end{cases}.$$

Using this transformation, the super – plane has the equation

$(\breve{2} \overline{\odot} \xi) \overline{\oplus} (\breve{2} \overline{\odot} \eta) \overline{\oplus} \zeta = \widetilde{\left(\frac{1}{2}\right)}$. (It is a hexagonal extra-plane in the new universe.)

Now, the joint point has the new coordinates $\left(\xi = 0, \eta = 0, \zeta = \widetilde{\left(\frac{1}{2}\right)}\right)$ is seen on the next figure:

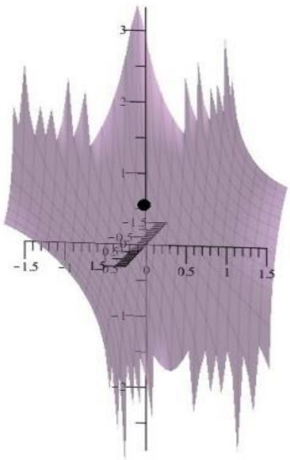

Fig. 9

Example 9. **Let** the extra-line be is the „w" axis of the rectangular Descartes coordinate-system and the extra – plane given by the equation

$$\breve{u}\overline{\oplus}\breve{v} = \breve{2}\overline{\odot}1 \Leftrightarrow \tanh u + \tanh v = 2\tanh 1\,,(u,v)\in\mathbb{R}^2\,,$$

are second–type missing elements of our universe. It is sufficient to cast a glance at the Fig. 5.

The following example shows one of essential characteristic of the extra geometry.

Example 10. The pair of extra-lines

$$\mathcal{L}(]\mathcal{B}_1,\mathcal{B}_2[) = \begin{cases} u = \overset{\frown}{\left(\frac{1}{4}\right)}\overline{\oplus}(\tau\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{26}}\right)}) \\ v = \overset{\frown}{\left(-\frac{1}{4}\right)}\overline{\oplus}(\tau\overline{\odot}\overset{\frown}{\left(\frac{3}{\sqrt{26}}\right)}) \\ w = \tau\overline{\odot}\overset{\frown}{\left(\frac{4}{\sqrt{26}}\right)} \end{cases} ,\ \overset{\frown}{\left(-\frac{\sqrt{26}}{4}\right)} < \tau < \overset{\frown}{\left(\frac{\sqrt{26}}{4}\right)}$$

And

$$\mathcal{L}(]\mathcal{B}_I,\mathcal{B}_{II}[) = \begin{cases} u = \overset{\frown}{\left(\frac{3}{4}\right)}\overline{\oplus}(\sigma\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{6}}\right)}) \\ v = \overset{\frown}{\left(-\frac{3}{4}\right)}\overline{\oplus}(\sigma\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{6}}\right)}) \\ w = \sigma\overline{\odot}\overset{\frown}{\left(\frac{2}{\sqrt{6}}\right)} \end{cases} ,\ \overset{\frown}{\left(-\frac{\sqrt{6}}{4}\right)} < \sigma < \overset{\frown}{\left(\frac{\sqrt{6}}{4}\right)}$$

has the following properties

a) They are situated on the same extra – plane
b) They are non-extra parallel extra – lines.
c) They have no joint point.

Verification.

Ad a)
It is easy to see that both extra – lines are situated on the extra plane described by the equation

$$u\overline{\oplus}v = w \quad ,(u,v,w)\in\mathbb{R}^3. \quad (\text{See Fig. 5.})$$

Ad b)

The border points of $\mathcal{L}(]\mathcal{B}_1,\mathcal{B}_2[)$ are $\mathcal{B}_1 = \left(0,\overset{\frown}{(-1)},\overset{\frown}{(-1)}\right)$ *and* $\mathcal{B}_2 = \left(\overset{\frown}{\left(\frac{1}{2}\right)},\overset{\frown}{\left(\frac{1}{2}\right)}\right),\breve{1}$ and the border point of $\mathcal{L}(]\mathcal{B}_I,\mathcal{B}_{II}[)$ are $\mathcal{B}_I = \left(\overset{\frown}{\left(\frac{1}{2}\right)},\overset{\frown}{(-1)},\overset{\frown}{(-1)}\right)$ *and* $\mathcal{B}_{II} = \left(\breve{1},\overset{\frown}{\left(-\frac{1}{2}\right)},\overset{\frown}{\left(\frac{1}{2}\right)}\right)$. By Definition 1 (see in [2]) we can see that $\mathcal{L}(]\mathcal{B}_1,\mathcal{B}_2[)$ and $\mathcal{L}(]\mathcal{B}_I,\mathcal{B}_{II}[)$ are non extra parallel extra – lines.

Ad c)
In search the joint point we find that the solution of equation system

$$\overset{\frown}{\left(\frac{1}{4}\right)}\overline{\oplus}(\tau\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{26}}\right)}) = \overset{\frown}{\left(\frac{3}{4}\right)}\overline{\oplus}(\sigma\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{6}}\right)})$$
$$\overset{\frown}{\left(-\frac{1}{4}\right)}\overline{\oplus}(\tau\overline{\odot}\overset{\frown}{\left(\frac{3}{\sqrt{26}}\right)}) = \overset{\frown}{\left(-\frac{3}{4}\right)}\overline{\oplus}(\sigma\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{6}}\right)})$$
$$\sigma\overline{\odot}\overset{\frown}{\left(\frac{2}{\sqrt{6}}\right)} = \sigma\overline{\odot}\overset{\frown}{\left(\frac{2}{\sqrt{6}}\right)}$$

is the pair $\tau = \overset{\frown}{\left(-\frac{\sqrt{26}}{2}\right)}$ and $\sigma = \overset{\frown}{\left(-\sqrt{6}\right)}$, only. Both parameters are outside the allowed parameter – domains, repectively. So, $\mathcal{L}(]\mathcal{B}_1,\mathcal{B}_2[)\cap\mathcal{L}(]\mathcal{B}_I,\mathcal{B}_{II}[) \neq \{\ \}\,(\mathit{empty\ set})$.

We remark, that the holders of extra – lines has the joint point $\left(\overset{\frown}{\left(-\frac{1}{4}\right)},\overset{\frown}{\left(-\frac{7}{4}\right)},\overset{\frown}{(-2)}\right)\notin\mathbb{R}^3$.

*Observation 3***.** In the extra geometry there exist evasive extra **-** lines such that they are situated on the same extra – plane.

*Definition 4***.** If the borders of two distinc exra – plane has a joint super – passage, then they are called extra parallel extra **-** planes.

Example 11. The extra – plane given by the equation

$$u\overline{\oplus}v = w \quad ,(u,v,w)\in\mathbb{R}^3 \quad ,(\text{see Fig. 2})$$

and the extra – plane given by the equation

$$\breve{3}\,\overline{\ominus}\,(\breve{2}\overline{\odot}u)\,\overline{\ominus}\,(\breve{2}\overline{\odot}v) = w \quad ,(u,v,w)\in\mathbb{R}^3 \quad ,(\text{see Fig. 8}),$$

are extra parallel extra – planes. Really, they have the joint (closed) border super – passage

$$\begin{cases} u = \overset{\frown}{\left(\frac{1}{2}\right)}\overline{\oplus}(\tau\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{2}}\right)}) \\ v = \overset{\frown}{\left(\frac{1}{2}\right)}\overline{\ominus}(\tau\overline{\odot}\overset{\frown}{\left(\frac{1}{\sqrt{2}}\right)}) \\ w = \breve{1} \end{cases} ,\ \overset{\frown}{\left(-\frac{1}{\sqrt{2}}\right)} \le \tau \le \overset{\frown}{\left(\frac{1}{\sqrt{2}}\right)}.$$

These extra parallel extra – planes are shown on the next figure

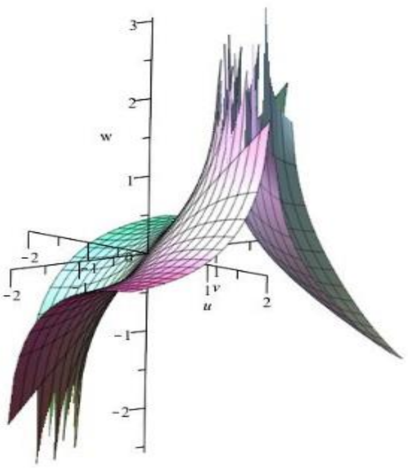

Fig.10

Their joint border super–passage is invisible on the Fig. 10 because it is situated in the border $\overline{\mathbb{R}^3} \setminus \mathbb{R}^3$. The form of the (open) border super – passage is visible on Fig. 3. (See the first quarter of the Descartes–coordinate system.)

Example 12. The quadragonal extra – plane (see (14) and the result of Exercise 3) given by the equation

$$u \overline{\oplus} v = 1 \overline{\odot}\, \breve{2} \quad ,(u,v) \in \mathbb{R}^2 \quad ,w \in \mathbb{R} \quad ,(\text{see Fig. } 5)$$

has four extra parallel extra–planes such that each of them contains the origo.

First extra – plane is given by the equations

$$u \overline{\oplus} v = 1 \overline{\odot}\, \breve{2} \overline{\odot}\, w \quad ,(u,v,w) \in \mathbb{R}^3$$

Or

$$w = \tanh^{-1} \frac{\tanh u + \tanh v}{2 \tanh 1} \quad ,-2 \tanh 1 < \tanh u + \tanh v < 2 \tanh 1$$

represented by the next figure

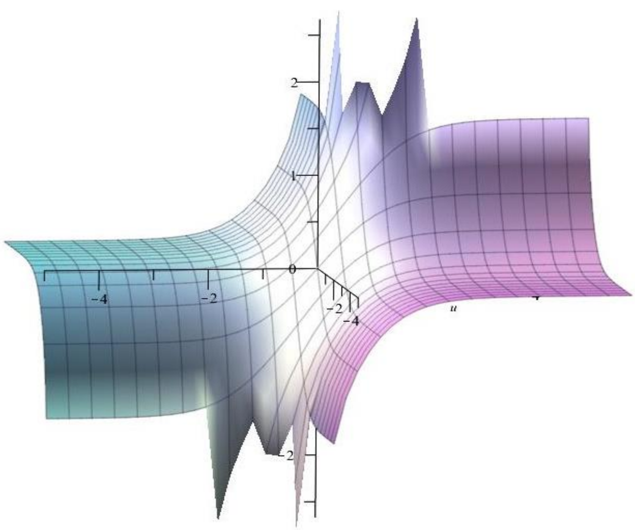

Fig. 11 (hexagonal extra-plane)

The joint (closed) border is a super–passage, a set of points $(u,v,w) \in \overline{\mathbb{R}^3} \setminus \mathbb{R}^3$

$$\begin{cases} u = 1 \overline{\oplus} (\tau \overline{\odot} \widetilde{\left(\frac{1}{\sqrt{2}}\right)}) \\ v = 1 \overline{\ominus} (\tau \overline{\odot} \widetilde{\left(\frac{1}{\sqrt{2}}\right)}) \\ w = \breve{1} \end{cases} , \widetilde{(-\sqrt{2})} \overline{\odot} (\breve{1} \overline{\ominus} 1) \le \tau \le \widetilde{(\sqrt{2})} \overline{\odot} (\breve{1} \overline{\ominus} 1).$$

Second extra – plane is the given by the equation

$$u \overline{\ominus} \left((\breve{2} \overline{\odot} 1) \overline{\ominus}\, \breve{1} \right) \overline{\odot}\, v = 0 \quad ,(u,v) \in \mathbb{R}^2 \quad ,w \in \mathbb{R}\,.$$

The joint (closed) border is a super–passage, a set of points $(u,v,w) \in \overline{\mathbb{R}^3} \setminus \mathbb{R}^3$

$$\begin{cases} u = (\breve{2} \overline{\odot} 1) \overline{\ominus}\, \breve{1} \\ v = \breve{1} \\ w = \tau \end{cases} , \widetilde{(-1)} \le \tau \le \breve{1}\,.$$

Third extra-plane is given by the equation

$$u \overline{\oplus} v \overline{\oplus} (1 \overline{\odot}\, \breve{2} \overline{\odot}\, w) = 0 \quad ,(u,v,w) \in \mathbb{R}^3.$$

The joint (closed) border is a super–passage, a set of points $(u,v,w) \in \overline{\mathbb{R}^3} \setminus \mathbb{R}^3$

$$\begin{cases} u = 1 \overline{\ominus} (\tau \overline{\odot} \widetilde{\left(\frac{1}{\sqrt{2}}\right)}) \\ v = 1 \overline{\oplus} (\tau \overline{\odot} \widetilde{\left(\frac{1}{\sqrt{2}}\right)}) \\ w = \widetilde{(-1)} \end{cases} , \widetilde{(-\sqrt{2})} \overline{\odot} (\breve{1} \overline{\ominus} 1) \le \tau \le \widetilde{(\sqrt{2})} \overline{\odot} (\breve{1} \overline{\ominus} 1)\,.$$

Fourth extra-plane is given by the equation

$$\left((\breve{2} \overline{\odot} 1) \overline{\ominus}\, \breve{1} \right) \overline{\odot}\, u = v \quad ,(u,v) \in \mathbb{R}^2 \quad ,w \in \mathbb{R}.$$

The joint (closed) border is a super–passage, a set of points $(u,v,w) \in \overline{\mathbb{R}^3} \setminus \mathbb{R}^3$

$$\begin{cases} u = \breve{1} \\ v = (\breve{2} \overline{\odot} 1) \overline{\ominus}\, \breve{1} \\ w = \tau \end{cases} , \widetilde{(-1)} \le \tau \le \breve{1}\,.$$

In the Euclidean geometry the relation between he planes is very simple:

- They have a joint line, only. (The planes cut each other.)
- They have no joint points. (They are parallel with each other.)

As it was already mentioned that the geometry of Multiverse is similar to the Euclidean geometry of our universe. (See Collection 1.) But the extra-geometry of our universe is more complicated: If two distinct extra–plane

- **A.)** have an only one joint line then the extra–planes cut each other,
- **B.)** have no joint points then we have three further cases:

  Case α) the extra – planes are extra parallel with each other (see Example 11),

  Case β) the holders of extra – planes have only one joint super –line outside $\overline{\mathbb{R}^3}$,

Case γ) the holders of extra –planes are super parallels.

*Definition 5***.** Extra – planes,

- in the of Case β) are called first – type missing extra - planes,
- in the of Case γ) are called second – type missing extra - planes

of our universe.

*Example 13***.** The extra – planes given by the equations

$(u \overline{\oslash} 1)\overline{\oplus}(v \overline{\oslash} 1) \overline{\oplus} (w \overline{\oslash} 1) = \breve{1} \quad ,(u,v,w) \in \mathbb{R}^3$ (Hexagonal extra-plane)

and

$(u \overline{\oslash} 1)\overline{\oplus}\left(v \overline{\oslash} \frac{1}{2}\right) \overline{\oplus} (w \overline{\oslash} 2) = \breve{1} \quad ,(u,v,w) \in \mathbb{R}^3$ (Pentagonal extra plane)

cut each other.

Really, they have only one joint extra – line a set of points $(u,v,w) \in \mathbb{R}^3$

$$\begin{cases} u = 1\overline{\oplus}\left(\tau\overline{\odot}\left(\frac{1}{\tanh 2}\,\breve{-}\,\frac{1}{\tanh\frac{1}{2}}\right)\right) \\ v = \tau\overline{\odot}\left(\frac{1}{\tanh 1}\,\breve{-}\,\frac{1}{\tanh 2}\right) \\ w = \tau\overline{\odot}\left(\frac{1}{\tanh\frac{1}{2}}\,\breve{-}\,\frac{1}{\tanh 1}\right) \end{cases} \quad , \overbrace{\left(\frac{1-\tanh 1}{\frac{1}{\tanh 2}-\frac{1}{\tanh\frac{1}{2}}}\right)} < \tau < \overbrace{\left(\frac{1}{\frac{1}{\tanh\frac{1}{2}}-\frac{1}{\tanh 1}}\right)}.$$

The cutting extra planes

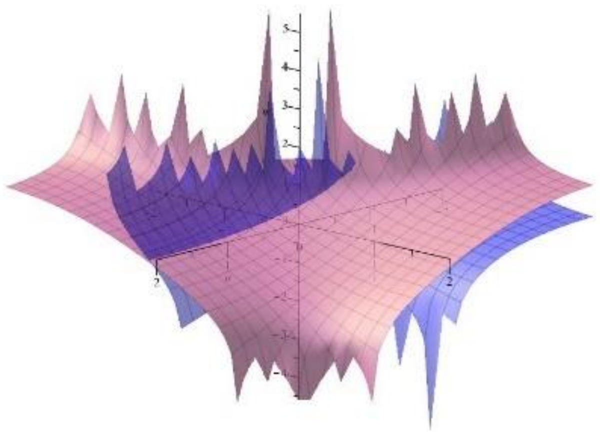

Fig. 12

*Example 14***.** The extra planes given by the equations

$$u\overline{\oplus}v \,\overline{\oplus}\, \breve{1} = w \quad ,(u,v,w) \in \mathbb{R}^3$$

and

$$\breve{3} \,\overline{\ominus}\, (\tilde{2} \,\overline{\odot}\, u) \,\overline{\ominus}\, (\tilde{2} \,\overline{\odot}\, v) = w \quad ,(u,v,w) \in \mathbb{R}^3$$

are first–type missing extra – planes.

Really, they have the holders

$$u\overline{\oplus}v \,\overline{\oplus}\, \breve{1} = w \quad ,(u,v,w) \in \breve{\mathbb{R}}^3$$

and

$$\breve{3} \,\overline{\ominus}\, (\tilde{2}\overleftrightarrow{\odot}u) \,\overline{\ominus}\, (\tilde{2}\overleftrightarrow{\odot}w) = w \quad ,(u,v,w) \in \breve{\mathbb{R}}^3 ,$$

respectively.

Clearly, these super–planes are non-super-parallel because they have the joint super-line

$$\begin{cases} u = \breve{\left(\frac{1}{3}\right)} \,\overline{\oplus}\, \tau \\ v = \breve{\left(\frac{1}{3}\right)} \,\overline{\ominus}\, \tau \quad , \tau \in \breve{\mathbb{R}} . \\ w = \breve{\left(\frac{5}{3}\right)} \end{cases}$$

We can see that

$$u\overline{\oplus}v \,\overline{\oplus}\, 1 = \left(\breve{\left(\frac{1}{3}\right)} \,\overline{\oplus}\, \tau\right) \overline{\oplus} \left(\breve{\left(\frac{1}{3}\right)} \,\overline{\ominus}\, \tau\right) \overline{\oplus}\, \breve{1} = \breve{\left(\frac{5}{3}\right)}$$

And

$$\breve{3} \,\overline{\ominus} \left(\breve{\left(\frac{1}{3}\right)} \,\overline{\oplus}\, \tau\right) \overline{\ominus} \left(\breve{\left(\frac{1}{3}\right)} \,\overline{\ominus}\, \tau\right) = \breve{\left(\frac{5}{3}\right)}$$

On the other hand this super – line is situated in the „height" $\breve{\left(\frac{5}{3}\right)}$ so, it is invisible in our universe. The first – type missing extra – planes are visible on the next figure.

*Example 15***.** The extra–planes given by the equations

$$u\overline{\oplus}v \,\overline{\oplus}\, 1 = w \quad ,(u,v,w) \in \mathbb{R}^3$$

and

$$u\overline{\oplus}v = w \quad ,(u,v,w) \in \mathbb{R}^3$$

are second–type missing extra–planes, because their holders

$$u\overline{\oplus}v \,\overline{\oplus}\, 1 = w \quad ,(u,v,w) \in \breve{\mathbb{R}}^3$$

and

$$u\overline{\oplus}v = w \quad ,(u,v,w) \in \breve{\mathbb{R}}^3 ,$$

are super-parallel super-planes.

REFERENCES

[1] István Szalay: Exploded and compressed numbers (enlargement of the universe, Parallel Universes, Extra Geometry), LAMBERT Academic Publishing, Saarbrücken, Germany, 2016, ISBN: 978-3-659-94402-4.

[2] I. Szalay and B. Szalay: Introduction into the extra geometry of the three-dimensional space I. European Journal of Engeneering Research and Sience (EJERS), Vol.5 No. 5. May 2020, 538-544.

[3] I. Szalay, B. Szalay: Extra Geometry I. Extra lines. International Journal of Engineering Research and Applications, (IJERA) Vol.8. Issue 1 (Part -1) January, 2018, 23 – 34.