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Modeling the effect of population and population augmented industrialization on forestry resources

Shyam Sundar, Niranjan Swaroop and Ram Naresh

***Abstract*—It is well known that rapid growth of population and population augmented industrialization has become a horrible threat for our environment causing the decline of forestry resources. We propose a mathematical model to study the effect of population and population density dependent industrialization on forestry resources. It is assumed that the industrialization grows logistically but its growth is further enhanced due to over population. It is shown that if the population and associated industrialization increase, the density of forestry resources decreases. Numerical simulation has also been conducted to support the analytical findings.**

***Index Terms*— Population; Industrialization; Forestry Resources; Stability Analysis; Numerical Simulation.**

I. Introduction

The main reason for the depletion of forestry resource is its excessive use to cater the need of continuously increasing human population for food and unabated industrialization in the name of development. Human population utilize forestry resources for their livelihood to meet the requirement of food, medicines etc. and for fuel, fodder for domesticated cattle. Clearing of forests in the name of industrialization and development includes construction of buildings, complexes, roads, resorts, industries, and other commercial activities. As the population increase, more and more forest resources are consumed in order to fulfill the growing need of enhanced population.

Several investigations have been made to study the effect of industrialization on forestry resources using mathematical models [1]-[7]. For example, [1] studied the effect of depletion of forestry biomass in a habitat due to pressure of industrialization on the survival of forestry biomass dependent wildlife species. They have shown that, under some conditions, the forest biomass density decreases due to an increase in industrialization pressure which leads to decrease in the density of wildlife species and it may even lead to extinction if the industrialization continues without control. Reference [2] studied the depletion of resource biomass due to industrialization and pollution by taking into account instantaneous and periodic emission of pollutants into the environment. Further the depletion of forestry resource due to population and population pressure augmented industrialization has also been studied by [3]. They have shown that the equilibrium density of resource biomass decreases as the equilibrium densities of population and industrialization increase. It is also found that the resource biomass may doom to extinction if the population pressure augmented industrialization is too large. Reference [4] have also studied the simultaneous effect of industrialization, population and pollution on the depletion of a renewable resource and have proved that if industrialization, population and pollution increase, the resource biomass may be driven to extinction.

Some mathematical investigations to comprehend the decline of forestry resources due to effect of pollutants/toxicants and human activities have also been made, [8]-[21]. In this regard, [9] studied the effects of toxicants (industrial emission) on the forestry resource and has shown that if effective steps are taken to control emission of these toxicants, the forestry resource biomass can be maintained at desirable level. Reference [17] have proposed a nonlinear mathematical model to study the adverse effects of increased population and pollution on forestry resources and found that resource biomass can be maintained at a desired level by conserving the forestry resource and by controlling the population growth and the emission of pollutants in the habitat. Reference [18] studied the depletion of forest resource biomass caused by population and the corresponding population pressure. It is observed that as the population density or population pressure increases, the cumulative density of forest resources decreases and it may even become extinct if the population pressure tends to be large enough. It is also suggested that by controlling the population pressure, using some economic incentives, the density of forest resources can be maintained at a desired equilibrium level. Reference [19], in their study, concluded that the depletion of forestry resource biomass due to human population, population pressure and industrialization can be controlled using sustained industrialization.

Gaseous pollutants emitted from industrial establishments adversely affect the forestry resources. Gaseous pollutants such as sulfur dioxide and nitrogen oxides alone as well as in the form of acid rain, as a result of combining with rain water or moisture present in the atmosphere, greatly affect the biodiversity and forestry resources, plants and crops etc. Pollutants emitted from industry pose greater hazards to health of human beings and other living organisms. Due to increased industrialization, population and large decay of forestry resources, precursor pollution is created in the environment which adversely affects the environment and the life of human beings. These effects have been studied by many researchers using mathematical models [22]-[28]. Reference [22] have developed a model to study the effects of industrialization and associated pollution on forest

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Shyam Sundar is with the Pranveer Singh Institute of Technology, Kanpur-209305, India (e-mail: ssmishra15@gmail.com), Corresponding author.

Niranjan Swaroop is with the Pranveer Singh Institute of Technology, Kanpur-209305, India (e-mail: saxena.niranjan@gmail.com).

Ram Naresh is with the School of Basic and Applied Sciences, Harcourt Butler Technical University, Kanpur-208002, India (e-mail: rnthbti@gmail.com).

resources and shown that wood-based industries deplete the forest resources both directly and indirectly but non-wood industries only deplete the forest resources indirectly. They concluded that with more and more industrialization the forest resources are severely affected and may become extinct. Reference [23] studied the effect of changed habitat due to increased population and industrialization on biological species and have shown that as the pressure of industrialization increases, the biomass density decreases. This decrease in biomass density leads to decrease the density of resource dependent species and may lead to their extinction if industrialization continues without any check. If effective steps are taken to conserve the resource biomass and to control the pressure of industrialization in the forested habitat, the survival of species can be ensured. Reference [26] investigated the effect of intermediate toxic product on plant biomass. They have shown that the toxic product, formed inside the plant body due to interaction of uptaken toxicant with sap (liquid present in plant body), is the main cause of decreasing growth rate and equilibrium level of plant biomass.

In view of the above, it is crucial to comprehend the role of industries, increasing due to overpopulation, in depleting the forestry resources. In this paper, we propose and analyze a nonlinear mathematical model to study the effect of population and population augmented industrialization on forestry resources assuming the growth of industries to be logistic.

II. The Model

To formulate the model, it is assumed that the population density $N(t)$, the biomass density $B(t)$ and the density of industrialization $I(t)$ grow logistically. It is assumed that the forestry biomass is utilized by human population to fulfill their needs leading to decline in biomass density where $s_1 > 0$ is the depletion rate coefficient of forestry resource due to population growth and the cumulative growth rate of the population is assumed to be $\nu_1 > 0$. Due to continuous increase in population, industries are established to fulfill their requirements by clearing forests to further decrease the density of forestry biomass with $\phi > 0$ as the depletion rate coefficient of forestry resource due to industrialization. Industrialization is being increased due to increase in population and because of the fact that after a certain level of industrialization their density will start decreasing due to mutual competition among them until equilibrium level is attained. Thus, we have assumed that $\delta > 0$ is the growth rate coefficient of the industrialization due to population and $u_1 > 0$ is its depletion rate coefficient due to crowding effect whereas the constant $u_0 > 0$ is the natural depletion rate coefficient of industrialization. The coefficients r, s and u are the intrinsic growth rates of N, B and I respectively with their respective carrying capacities as K, L and M. All the constants, taken in the model system, are assumed to be positive.

Thus, the system is assumed to be governed by the following nonlinear ordinary differential equations,

$$\frac{dN}{dT} = rN\left(1-\frac{N}{K}\right)+\nu_1 s_1 BN \tag{1}$$

$$\frac{dB}{dt} = sB\left(1-\frac{B}{L}\right)-s_1 BN-\phi BI \tag{2}$$

$$\frac{dI}{dt} = uI\left(1-\frac{I}{M}\right)+\delta NI-u_0 I-u_1 I^2 \tag{3}$$

$$N(0)\geq 0, B(0)\geq 0, I(0)\geq 0$$

III. Equilibrium Analysis

The model system has the following non-negative equilibria,

(i) $E_0(0,0,0)$, the trivial equilibrium

(ii) $E_1(0,0,I_1)$ where $I_1 = \dfrac{u-u_0}{\frac{u}{M}+u_1}$

(iii) $E_2 = (0,L,0)$

(iv) $E_3 = (0,B_3,I_3)$ where $I_3 = \dfrac{u-u_0}{\frac{u}{M}+u_1}$ and

$$B_3 = L-\frac{L\phi}{s}\left(\frac{u-u_0}{\frac{u}{M}+u_1}\right)$$

(v) $E_4(K,0,0)$

(vi) $E_5(K,0,I_5)$ where $I_5 = \dfrac{u-u_0+\delta K}{\frac{u}{M}+u_1}$

(vii) $E_6(N_6,B_6,0)$ where $N_6 = \dfrac{r+\nu_1 s_1 L}{\frac{r}{K}+\frac{\nu_1 s_1^2 L}{s}}$,

$B_6 = \dfrac{rL(s-Ks_1)}{rs+K\nu_1 s_1^2 L}$ and $s-s_1K>0$.

(viii) $E^*(N^*,B^*,I^*)$

Existence of all equilibria except E^* is obvious. We now prove the existence of E^* as follows.

Equilibrium values of different variables in E^* are given by the following algebraic equations,

$$r\left(1-\frac{N}{K}\right)+\nu_1 s_1 B = 0 \tag{4}$$

$$s\left(1-\frac{B}{L}\right)-s_1 N-\phi I = 0 \tag{5}$$

$$u\left(1-\frac{I}{M}\right)+\delta N-u_0-u_1 I = 0 \tag{6}$$

Now from (6) we get

$$I = \frac{u-u_0+\delta N}{\frac{u}{M}+u_1} = g(N) \tag{7}$$

In view of (7), (5) can be written as,

$$B = \frac{L}{s}\left[s-s_1 N-\phi g(N)\right] = f(N) \tag{8}$$

Now (4) can be written as

$$F(N)=r\left(1-\frac{N}{K}\right)+v_1 s_1 f(N) \tag{9}$$

from which we get

$$F(0)=r+v_1 s_1 \frac{L}{s}\left(s-\frac{\phi(u-u_0)}{\frac{u}{M}+u_1}\right) \tag{10}$$

From above, it is clear that

$$F(0)>0 \text{ if } \phi<\frac{s\left(\frac{u}{M}+u_1\right)}{u-u_0} \tag{11}$$

Again from (9) we get

$$F(K)=v_1 s_1 \frac{L}{s}\left[s-s_1 K-\frac{\phi(u-u_0+\delta K)}{\frac{u}{M}+u_1}\right]$$

and thus $F(K)<0$ if $\phi>\dfrac{s\left(\frac{u}{M}+u_1\right)}{u-u_0+\delta K}$ (12)

From (11) and (12) it is clear that N^* exists if

$$\frac{s\left(\frac{u}{M}+u_1\right)}{u-u_0+\delta K}<\phi<\frac{s\left(\frac{u}{M}+u_1\right)}{u-u_0} \tag{13}$$

This implies that the equilibrium level can be attained only if the rate of depletion of forestry biomass (i.e. ϕ) due to industrialization satisfies condition (13) and thus equilibrium E^* exists provided the condition (13) is satisfied.

Again, from (9) we get

$$F'(N)=-\frac{r}{K}-v_1 s_1 \frac{L}{s}\left(s_1+\phi\frac{\delta}{\frac{u}{M}+u_1}\right)<0 \tag{14}$$

which implies that the equilibrium E^* is unique.

The region of attraction of the model system (1) – (3) is given in the following Lemma.

A. Lemma

The set $\Omega=\{(N^*,B^*,I^*)\in R^3: 0\le N\le N_m, 0\le B\le L,$

$0\le I\le \frac{M}{u}\left(u-u_0+\delta N_m\right)\}$

is the region of attraction for all solutions of the model system (1) – (3) initiating in the interior of positive octant, where $N_m=\frac{K}{r}\left(r+v_1 s_1 L\right)$

Proof

From (1), we note that

$$\frac{dB}{dt}=sB\left(1-\frac{B}{L}\right)-s_1 BN-\phi BI$$

$$\le sB\left(1-\frac{B}{L}\right)$$

Thus, we get $0\le B\le L$

Similarly, from (2) and (3), we get,

$$0\le N\le \frac{K}{r}\left(r+v_1 s_1 L\right)=N_m \text{ and } I\le \frac{M}{u}\left(u-u_0+\delta N_m\right)$$

B. Variation of different variables with suitable parameters

(i) *Variation of* B *with* s_1,ϕ *and* N *with* s_1

We have from (7) and (8),

$$B=\frac{L}{s}\left(s-s_1 N-\phi\frac{(u-u\ \ +\delta N)}{\frac{u}{M}+u_1}\right) \tag{15}$$

From (4) we get

$$N=(r+v_1 s_1 B)\frac{K}{r} \tag{16}$$

and from (15) and (16), we get,

$$B\left(1+\left(s_1+\frac{\phi\delta}{\frac{u}{M}+u_1}\right)\frac{KL}{rs}v_1 s_1\right)$$

$$=\frac{L}{s}\left(s-\frac{\phi(u-u_0)}{\frac{u}{M}+u_1}-\left(s_1+\frac{\phi\delta}{\frac{u}{M}+u_1}\right)K\right) \tag{17}$$

Differentiating (17) with respect to ϕ we get

$$\frac{dB}{d\phi}A+B\frac{\delta v_1 s_1}{\frac{u}{M}+u_1}\frac{KL}{rs}=-\frac{L}{s}\left(\frac{u-u_0+\delta K}{\frac{u}{M}+u_1}\right) \tag{18}$$

where $A=1+\left(s_1+\frac{\phi\delta}{\frac{u}{M}+u_1}\right)\frac{KL}{r}v_1 s_1$

From (18) we get

$$\frac{dB}{d\phi}=-\frac{B}{A}\frac{\delta v_1 s_1}{\frac{u}{M}+u_1}\frac{KL}{rs}-\frac{L}{s}\left(\frac{u-u_0+\delta K}{\frac{u}{M}+u_1}\right)<0$$

Similarly we can show that $\frac{dB}{ds_1}<0$ and $\frac{dN}{ds_1}>0$

This implies that the density of forestry biomass decreases as the depletion rate coefficient of forestry resource due to industrialization and population growth increase. This, in turn, increases the population growth. Thus, there must be equilibrium among population, forestry biomass and industrialization.

IV. Stability Analysis

A. Local Stability

In this section, we perform stability analysis for the feasibility of the equilibrium points of the model system (1)-(3). The Jacobian matrix M for the model system (1)-(3) is given as follows,

$$M = \begin{bmatrix} r - 2r\dfrac{N}{K} + \nu_1 s_1 B & \nu_1 s_1 N & 0 \\ -s_1 B & s - \dfrac{2sB}{L} - s_1 N - \phi I & -\phi B \\ \delta I & 0 & u - 2u\dfrac{I}{M} + \delta N - u_0 - 2u_1 I \end{bmatrix}$$

From the above matrix, we note that,

(i) Equilibrium $E_0(0,0,0)$ is a unstable node as the eigenvalues of the Jacobian matrix M corresponding to E_0 are all positive.

(ii) Equilibrium $E_1(0,0,I_1)$ is a saddle point which is unstable as two eigenvalues of the Jacobian matrix M corresponding to E_1 are positive and one is negative.

(iii) Equilibrium $E_2(0,L,0)$ it is a saddle point which is unstable as two eigenvalues of the Jacobian matrix M corresponding to E_2 are positive and one is negative.

(iv) Equilibrium $E_3(0,B_3,I_3)$ is unstable as one eigenvalue of the Jacobian matrix M corresponding to E_3 is positive.

(v) Equilibrium $E_4(K,0,0)$ is unstable as two eigenvalues of the Jacobian matrix M corresponding to E_4 are positive and one is negative.

(vi) Equilibrium $E_5(K,0,I_5)$ is unstable as one eigenvalue of the Jacobian matrix M corresponding to E_5 is positive.

(vii) Equilibrium $E_6(N_6,B_6,0)$ is unstable as one eigenvalue of the Jacobian matrix M corresponding to E_6 is positive.

The eigenvalues of Jacobian matrix M corresponding to equilibrium $E^*(N^*,B^*,I^*)$ are given by,

$$t^3 + a_1 t^2 + a_2 t + a_3 = 0 \qquad (19)$$

where,

$$a_1 = \frac{r}{K}N^* + \frac{s}{L}B^* + \left(\frac{u}{M} + u_1\right)I^*$$

$$a_2 = \frac{r}{K}N^* \frac{s}{L}B^* + \frac{s}{L}B^*\left(\frac{u}{M} + u_1\right)I^* + \frac{r}{K}N^*\left(\frac{u}{M} + u_1\right)I^* + \nu_1 s_1^2 B^* N^*$$

$$a_3 = \frac{r}{K}N^* \frac{s}{L}B^*\left(\frac{u}{M} + u_1\right)I^* + \nu_1 s_1^2 B^* N^* I^*\left(\frac{u}{M} + u_1\right) + \nu_1 s_1 \delta\phi N^* B^* I^*$$

Since all the coefficients (i.e. $a_i, i = 1,2,3$) are positive, therefore all the eigenvalues of Jacobian matrix M corresponding to E^* are either negative or have negative real part if $a_1 a_2 > a_3$. This implies that E^* is locally stable.

Theorem 1

The equilibrium E^* is locally stable if $a_1 a_2 > a_3$.

B. Nonlinear Stability

To establish the nonlinear stability behavior of E^*, we consider the following positive definite function about E^* [29],

$$V = m_1\left(N - N^* - N^* \log\frac{N}{N^*}\right) - m_2\left(B - B^* - B^* \log\frac{B}{B*}\right) + m_3\left(I - I^* - I^* \log\frac{I}{I*}\right) \qquad (20)$$

Differentiating (20) with respect to $'t'$ along the model system (1) – (3) we get,

$$\frac{dV}{dt} = m_1(N - N^*)\frac{1}{N}\frac{dN}{dt}\frac{dI}{dt} + m_2(B - B^*)\frac{1}{B}\frac{dB}{dt} + m_3(I - I^*)\frac{1}{I}\frac{dI}{dt} \qquad (21)$$

$$\frac{dV}{dt} = m_1(N - N^*)\left[-\frac{r}{K}(N - N^*) + \nu_1 s_1(B - B^*)\right] + m_2(B - B^*)\left[-\frac{s}{L}(B - B^*) - s_1(N - N^*) - \phi(I - I^*)\right] + m_3(I - I^*)\left[-\left(\frac{u}{M} + u_1\right)(I - I^*) + \delta(N - N^*)\right]$$

$$\frac{dV}{dt} = -m_1\frac{r}{K}(N - N^*)^2 - m_2\frac{s}{L}(B - B^*)^2 - m_3\frac{u}{M}(I - I^*)^2 - m_3 u_1(I - I^*)^2 + (m_1\nu_1 s_1 - m_2 s_1)(B - B^*)(N - N^*) - m_2\phi(B - B^*)(I - I^*) + m_3\delta(N - N^*)(I - I^*)$$

For $\dfrac{dV}{dt}$ to be negative definite, the following sufficient conditions must be satisfied,

$$(m_1\nu_1 s_1 - m_2 s_1)^2 < m_1 m_2\frac{rs}{KL} \qquad (22)$$

$$(m_2\phi)^2 < m_2 m_3\frac{s}{L}\left(\frac{u}{M} + u_1\right) \qquad (23)$$

$$(\delta m_3)^2 < m_1 m_3\frac{r}{K}\left(\frac{u}{M} + u_1\right) \qquad (24)$$

After maximizing L.H.S. and minimizing R.H.S. and choosing $m_1 = 1$ and $m_2 = \nu_1$, we get the following condition,

$$\delta^2\phi^2\nu_1 < \frac{rs}{KL}\left(\frac{u}{M} + u_1\right)^2 \qquad (25)$$

Thus, under the above condition, $\dfrac{dV}{dt}$ will be negative definite showing that V is a Lyapunov's function and hence the following theorem is established.

Theorem 2

The equilibrium E^* is nonlinearly stable inside the region of attraction Ω, if the condition (25) holds.

Remark

This theorem implies that if the rate of depletion of forestry resources due to industrialization or the growth rate of industrialization due to increase in population is small then the possibility of satisfying these conditions is more plausible. Thus, in order to maintain equilibrium, we should establish harmony among population, forestry biomass and industrialization.

V. Numerical Simulation

In this section, we perform some numerical simulations to study the local and nonlinear stability behaviour of equilibria and feasibility of the model system (1) – (3) numerically using MAPLE 18 by choosing the following set of parameter values,

$r = 1,\ K = 1000,\ \nu_1 = .2,\ s_1 = 0.0001,\ s = 1.2,\ L = 1500,$
$\phi = 0.0005,\ u = 0.1,\ M = 10,\ \delta = 0.0003,\ u_0 = 0.01,$
$u_1 = 0.02$

The equilibrium values of different variables in E^* corresponding to above data are given by

$N^* = 1027.265927,\ B^* = 1363.296347,\ I^* = 13.2726592$

The eigenvalues of the Jacobean matrix corresponding to $E^*(N^*, B^*, I^*)$ for the model system (1)-(3) are $-1.0588 + 0.0413i$, $-1.0588 - 0.0413i$, -0.3983 which are either negative or have negative real part, as a result the interior equilibrium $E^*(N^*, B^*, I^*)$ is locally asymptotically stable.

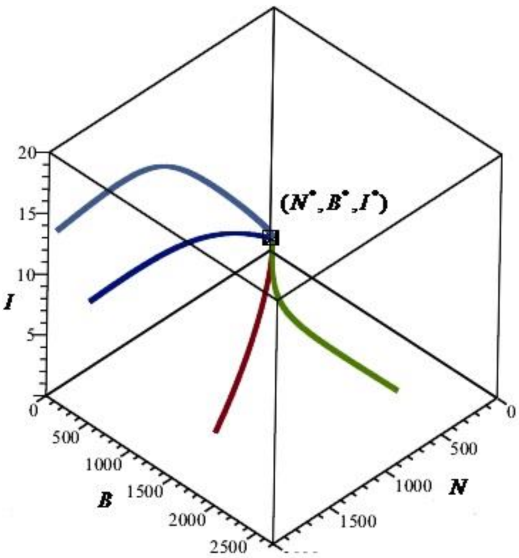

Fig. 1. Nonlinear Stability in $N-B-I$ plane

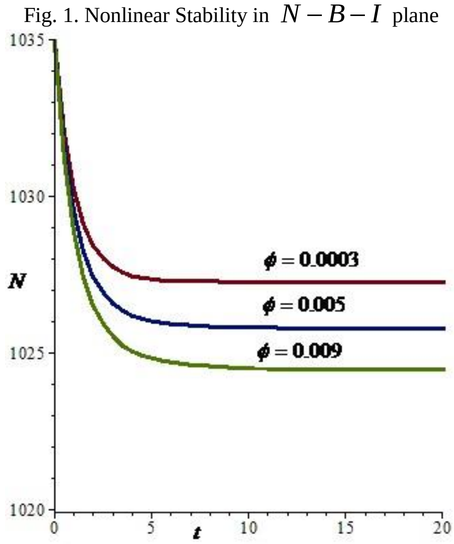

Fig. 2. Variation of population N with time t for different values of ϕ

The nonlinear stability behaviour of E^* is shown in the figure 1. This figure depicts that the solution trajectories that start at any point within the region of attraction approach to equilibrium E^*. In figure 2, the variation of population with time for different values of depletion rate coffiecient ϕ of forestry resource due to industrialization is plotted. It is seen that the population N decreases with increase in ϕ. Thus with decrease in the forestry resources due to industrialization, the growth of population gets severely affected. This implies that for survival of population, the depletion of forestry resources due to industrialization be minimized. The depletion of forestry resource biomass B due to industrialization is shown in figure 3. From this figure, we note that forestry resource biomass B decreases as the depletion rate coefficient ϕ due to industrialization increases. This indicates that clearing of forest biomass required for industrial purposes should not be unabated. The effect of enhanced industrialization due to population growth on forestry biomass is shown in figure 4., from which it is found that the forest biomass B decreases as the rate of industrialization δ due to population increases. The variation of industrialization density with time for different values of δ, the growth rate coefficient of industrialization due to population is depicted in figure 5. It is seen that increase in industrialization is observed with increase in the values of δ.

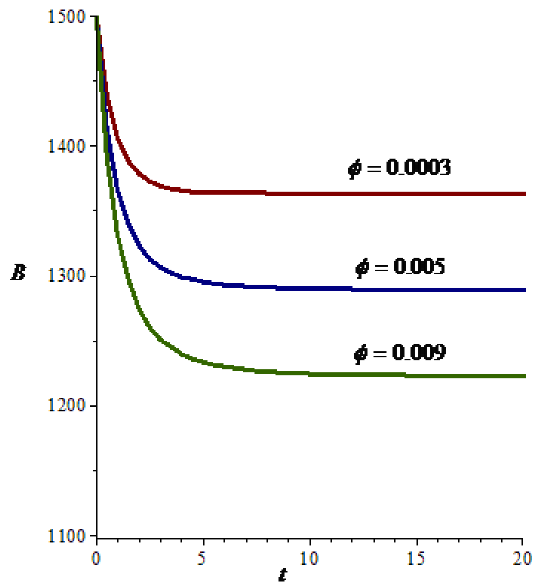

Fig. 3. Variation of forestry resource biomass B with time t for different values of ϕ

From the above, it can be speculated that for survival of population, the depletion of forestry resource biomass due to industrialization be kept at minimum so that the equilibrium is maintained.

VI. Conclusion

A nonlinear mathematical model has been proposed and analyzed to study the depletion of forestry resource biomass due to human population and population augmented industrialization. Existence of interior equilibrium is established and its local as well as nonlinear stability studied for the model system. It is shown that the density of forestry biomass decreases as the depletion rate coefficient of forestry resource due to industrialization increases which ultimately affects the population growth. Numerical simulations validate our analytical findings.

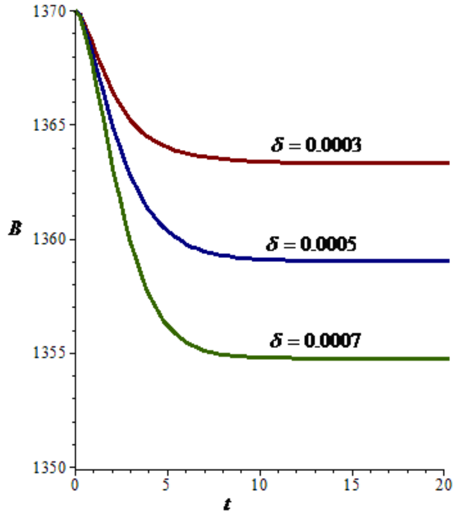

Fig. 4. Variation of forestry resource biomass B with time t for different values of δ

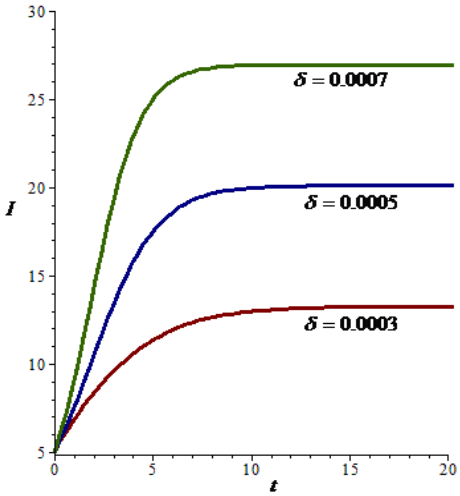

Fig. 5.Variation of industrialization I with time t for different values of δ

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