Unified Vector Multiplication Approach for Calculating Convolution and Correlation

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***Abstract* — This paper is a theoretical analysis of discrete time convolution and correlation and to introduce a unified vector multiplication approach for calculating discrete convolution and correlation, both of which are important concepts in the design and analysis of signals and systems and are usually encountered in the first course in signals and systems analysis. There are software tools for calculating them, however, it is important to learn now to compute them by hand. Several methods have been proposed to compute them by hand, most of which can be very involving. However, a closer look at the concepts reveal that the convolution and correlation sums are actually vector multiplication with diagonalwise addition and for finite sequences, can be computed by hand the same way. The method is also extended to N-point circular convolution. The method also makes it clearer to see the similarities and differences between convolution and correlation.**

***Index Terms* — Convolution, correlation, N-point circular convolution, vector multiplication.**

I. Introduction

Convolution and correlation are basic foundations in the analysis and design of signals and systems. The convolution and correlation of two signals are usually computed using summation for discrete-time (DT) signals. DT convolution is usually referred to as the *convolution sum*. For DT signals, there are two types of convolution-*linear convolution* (which is basic), and *N-point circular convolution*. Correlation can be computed as *crosscorrelation* (between two different signals), or *autocorrelation* (a signal with itself). A first approach to understanding linear convolution is by computing the convolution sum of two finite length sequences of 1-dimension and it has been described in most of the popular textbooks in signals and systems analysis e.g. [1]–[6]. Correlation is discussed in [1], [3] and [4].

For convolution the solution methods usually presented in the books are *analytical convolution*, *graphical convolution*, and the *Z-transform approach*. For correlation, analytical and graphical methods are also presented but by using finite length sequences. Analytical convolution is more generic in that it can obtain a closed-form solution when the lengths of the sequences are unknown. In graphical convolution, the graphical representation of both signals to be convolved are drawn and manipulated to obtain the convolution sum.

None of the books actually present the convolution and correlation sums as the vector multiplication of both signals with diagonal addition.

It is acknowledged here that it is indispensable to understand analytical convolution because that is the only method that can obtain a closed form solution. Moreover, the vector multiplication approach presented here is not to prove that it is the superior method to any of the other approaches, rather it is an alternative method to arrive at the same solution. It is left for the readers to compare and decide which approach(es) to use.

In [7], it is shown that the discrete convolution *of finite length sequences* is analogous to polynomial multiplication. However, it was presented as a *novel method for calculating the convolution sum of two finite length sequences*. In this paper, an attempt is made to generalize and show that whichever way the convolution sum is calculated, and *whether the sequences are finite or not*, the convolution sum actually corresponds to the vector multiplication of both signals followed by diagonal addition.

Also, most of the reviewed textbooks explain that convolution in the time domain is multiplication in the Z-domain. Therefore, one may obtain the convolution sum by first obtaining the Z-transforms of both signals before performing the multiplication which results in the convolution sum. However, it will be shown here that for finite length signals, one can directly perform the vector multiplication of both signals in the time domain without taking the Z-transforms and obtain the same result as the Z-transform approach.

The rest of the paper is organized as follows. In Section II the convolution sum is described in terms of vector multiplication by using some examples to show that the sum obtained by the analytical, graphical and Z-transform approaches can be obtained by *vector multiplication with left diagonal addition*. In Section III correlation is described in terms of *vector multiplication with right diagonal* addition by using some examples. In Section IV the vector multiplication method is extended to N-point circular convolution. In Section V the commutativity property is analyzed for convolution and correlation in terms of the vector multiplication approach. Section VI summarizes the paper.

II. Convolution As Vector Multiplication With Left Diagonal Addition

To show that the convolution sum is vector multiplication with left diagonal addition several examples will be presented from the reviewed books. The analytical, graphical and Z-transform solutions will be presented (or references will be made to solutions in pages in the books). Then the vector

Submitted on June 05, 2021.
Published on June 26, 2021.
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multiplication approach will be presented, and the solutions compared.

A. Finite Sequence Example

Consider the finite sequences of Example 2.3.2 in [4] (pp. 75-76) which is as follows. Compute the convolution of $h[k] = [1, \hat{2}, 1, -1]$ and $x[k] = [\hat{1}, 2,3,1]$. The ˆ indicates the $k = 0$ sample. Linear convolution is usually represented by the infinite summation:

$$y[k] = x[k] * h[k] = \sum_{m=-\infty}^{\infty} x[m]\, h[k-m] \qquad (1)$$

From $h[k]$ and $x[k]$, $-1 \leq k \leq 5$, and since $x[k]$ is causal, the analytical solution is as follows:

$$y[k] = \sum_{m=0}^{3} x[m]h[k-m]$$

$$y[-1] = x[0]h[-1] + x[1]h[-2] = 1$$

$$y[0] = x[0]h[0] + x[1]h[-1] = 2 + 2 = 4$$

$$y[1] = x[0]h[1] + x[1]h[0] + x[2]h[-1] = 2 + 4 + 2 = 8$$

$$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0] + x[3]h[-1] = 8$$

$$y[3] = x[1]h[2] + x[2]h[1] + x[3]h[0] = -2 + 3 + 2 = 3$$

$$y[4] = x[2]h[2] + x[3]h[1] = -3 + 1 = -2$$

$$y[5] = x[3]h[2] = -1$$

The graphical solution is presented in [4] (p. 76) and occupies a full page. The solution obtained by vector multiplication with left diagonal addition is as follows.

First, the signals to be convolved $x[k], h[k]$ are represented as row vectors. Then the convolution sum is defined as

$$y[k] = x[k] * h[k] = \sum_{k=-\infty}^{\infty} \measuredangle\, [k] h^T x \qquad (2)$$

Equivalently:

$$y[k] = h[k] * x[k] = \sum_{k=-\infty}^{\infty} \measuredangle\, [k] x^T h$$

where $\sum_{k=-\infty}^{\infty} \measuredangle\, [k]$ indicates the *left diagonal* sums of the matrix obtained from $h^T x$ or $x^T h$. Let

$$y' = h^T x$$

Then

$$y' = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 1 \\ -1 & -2 & -3 & -1 \end{bmatrix}$$

Taking left diagonal sums of y' gives

$$y[n] = [1, \hat{4}, 8, 8, 3, -2, -1]$$

Observe that each left diagonal sum corresponds to each $y[n]$ of the analytical solution.

One may easily obtain y' by using a scientific calculator or software. Then what is left is simply additions. Otherwise, one may perform the multiplications and addition by hand using the tabular form shown below.

	$\hat{1}$	2	3	1	$x[k]$ / $h[k]$
	1	2	3	1	1
1	2	4	6	2	$\hat{2}$
4	1	2	3	1	1
8	−1	−2	−3	−1	−1
$y[k]$	8	3	−2	−1	

$$y[n] = [1, \hat{4}, 8, 8, 3, -2, -1]$$

B. Non-finite Sequence Example

Consider Example 9.6 from [1] (pp. 588-589), which is as follows. Determine the convolution sum $c[k] = f[k] * g[k]$ for the signals

$$f[k] = (0.8)^k u[k] \text{ and } g[k] = (0.3)^k u[k].$$

The analytical closed form solution presented in [1] is

$$c[k] = 2[(0.8)^{k+1} - (0.3)^{k+1}]u[k]$$

Obtaining $c[k]$ samples for $k = 0, \cdots, 25$ ($c[k] \approx 0$ for $k > 25$) yields the sequence:

$c[k]$ = [1, 1.1, 0.97, 0.8, 0.65, 0.52, 0.42, 0.34, 0.27, 0.21, 0.17, 0.14, 0.11, 0.09, 0.07, 0.06, 0.05, 0.04, 0.03, 0.023, 0.02, 0.02, 0.012, 0.0094, 0.0076, 0.006].

The graphical solution is presented in Example 9.8 [1] (p. 593) with the same result as $c[k]$ above. It occupies a full page. The final result is shown in Fig. 1.

To obtain the vector multiplication solution, finite numeric samples of $f[k]$ (for $k = 0, \cdots, 20$) and $g[k]$ (for $k = 0, \cdots, 3$) are taken. Table I shows the multiplication and solution obtained. Evidently, the result is the same as the analytical solution (which is closed form) and the graphical solution. This is the first paper where a closed form solution is validated numerically by vector multiplication of samples of the original sequences.

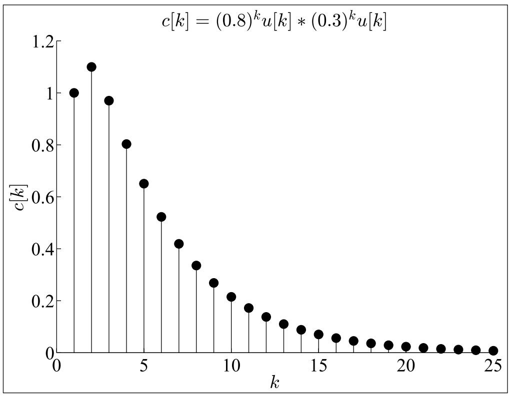

Fig. 1. The convolution sum $\boldsymbol{c[k] = f[k] * g[k]}$.

TABLE I: C[K] BY VECTOR MULTIPLICATION

	0	1	2	3	4	5	6	7	8	9	10	k
	$\hat{1}$	0.8	0.64	0.51	0.41	0.33	0.26	0.21	0.17	0.13	0.11	$f[k]$ / $g[k]$
	1	0.8	0.64	0.51	0.41	0.33	0.26	0.21	0.17	0.13	0.11	$\hat{1}$
$\hat{1}$	0.3	0.24	0.18	0.15	0.12	0.09	0.09	0.06	0.05	0.039	0.033	0.3
1.1	0.09	0.07	0.05	0.05	0.04	0.03	0.03	0.02	0.015	0.012	0.009	0.09
0.97	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.006	0.005	0.004	0.003	0.03
$c[k]$	0.79	0.64	0.52	0.4	0.34	0.27	0.21	0.17	0.136	0.109	0.087	

	11	12	13	14	15	18	17	18	19	20	k
	0.086	0.069	0.055	0.044	0.035	0.028	0.023	0.018	0.014	0.012	$f[k]$ / $g[k]$
	0.086	0.069	0.055	0.044	0.035	0.028	0.023	0.018	0.014	0.012	$\hat{1}$
$\hat{1}$	0.027	0.021	0.018	0.012	0.012	0.009	0.006	0.006	0.003	0.003	0.3
1.1	0.008	0.006	0.005	0.0036	0.0036	0.0027	0.0018	0.0018	0.0009	0.0009	0.09
0.97	0.003	0.002	0.0018	0.0012	0.0012	0.0009	0.0006	0.0006	0.0003	0.0003	0.03
$c[k]$	0.071	0.055	0.0454	0.0368	0.0279	0.0227	0.0174	0.0045	0.0012	0.0003	

The small differences in latter values are due to approximations.

In addition to the analytical and graphical solutions usually presented in the books, a *sliding tape method* as an alternative to graphical convolution is presented in [1] as Example 9.9 (pp. 594-595). The tabularized vector multiplication solution is shown below.

	$\hat{0}$	1	2	3	4	5	$f[k]$ / $g[k]$
	0	1	2	3	4	5	$\hat{1}$
0	0	1	2	3	4	5	1
1	0	1	2	3	4	5	1
3	0	1	2	3	4	5	1
6	0	1	2	3	4	5	1
10	0	1	2	3	4	5	1
$c[k]$	15	15	14	12	9	5	

$$c[k] = [\hat{0}, 1, 3,6,10,15,15,14,12,9,5]$$

The solution is the same as that obtained from the sliding tape method. In addition, [1] (p. 590) provides a *table of convolution sums* to aid in the calculation of analytical convolution.

C. Z-transform Example

Next, it is shown that for finite sequences (or unless a closed form solution is desired), one does not need to apply the Z-transform to a pair of sequences before multiplication in order to simplify the computation of the convolution sum. Rather, one can apply vector multiplication directly. To this end, consider Example 3.2.9 in [4] (p. 165) which is as follows. Compute the convolution $x[n]$ of the signals:

$$x_1[n] = [\hat{1}, -2,1];\ x_2[n] = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & \text{otherwise} \end{cases}$$

The solution using the Z-transform approach presented in [4] is as follows:

First, the Z-transform of both signals is calculated

$$X_1(z) = 1 - 2z^{-1} + z^{-2}$$

$$X_2(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}$$

Then both transforms are multiplied to obtain

$$X(z) = X_1(z)X_2(z)$$

as follows:

1	$-2z^{-1}$	$+z^{-2}$					
1	$+z^{-1}$	$+z^{-2}$	$+z^{-3}$	$+z^{-4}$	$+z^{-5}$		
1	$+z^{-1}$	$+z^{-2}$	$+z^{-3}$	$+z^{-4}$	$+z^{-5}$		
	$-2z^{-1}$	$-2z^{-2}$	$-2z^{-3}$	$-2z^{-4}$	$-2z^{-5}$	$-2z^{-6}$	
		$+z^{-2}$	$+z^{-3}$	$+z^{-4}$	$+z^{-5}$	$+z^{-6}$	$+z^{-7}$
1	$-z^{-1}$	$+0z^{-1}$	$+0z^{-3}$	$+0z^{-4}$	$+0z^{-5}$	$-z^{-6}$	$+z^{-7}$

Finally, the convolution sum is obtained as the coefficients of $X(z)$

$$x[n] = [\hat{1}, -1, 0,0,0,0, -1,1]$$

The tabularized solution obtained from vector multiplication and left diagonal addition is shown below.

	$\hat{1}$	1	1	1	1	1	$x_2[k]$ / $x_1[k]$
	1	1	1	1	1	1	$\hat{1}$
1	−2	−2	-2	−2	−2	−2	−2
−1	1	1	1	1	1	1	1
$x[n]$	0	0	0	0	−1	1	

$$x[n] = [\hat{1}, -1, 0,0,0,0, -1,1]$$

From the examples seen so far one can make an informed comparison between the vector multiplication approach and the other approaches to calculating the convolution sum, in terms of the solutions obtained and the complexities involved.

III. CORRELATION SUM AS VECTOR MULTIPLICATION WITH RIGHT DIAGONAL ADDITION

The correlation of two signals is a measure of how much both signals are similar. Continuous time (CT) correlation was considered in [1] (pp. 177-182), DT correlation and circular correlation in [3] (pp. 430-459), and DT correlation in [4] (pp. 116-128). For two signals $y[n]$, $x[n]$ both with finite energy, the crosscorrelation sequence is

$$r_{xy}[k] = \sum_{m=-\infty}^{\infty} x[m]y[m-k]\,, k = 0, \pm 1, \pm 2 \quad (3)$$

or

$$r_{xy}[k] = \sum_{m=-\infty}^{\infty} x[m+k]y[m]\,, k = 0, \pm 1, \pm 2, \cdots$$

where xy is the cross correlation and k is time shift or lag.

The *crosscorrelation inverse* inverts the roles of the former sequence $r_{xy}[k]$ and is given as:

$$r_{yx}[k] = \sum_{m=-\infty}^{\infty} y[m]x[m-k]\,, k = 0, \pm 1, \pm 2 \qquad (4)$$

or

$$r_{yx}[k] = \sum_{m=-\infty}^{\infty} y[m+k]x[m]\,, k = 0, \pm 1, \pm 2$$

The vector multiplication with diagonal addition approach presented for convolution is extended to correlation. The finite sequence example in section II will be repeated here in order to show the difference between correlation and convolution. The solution applies to the other examples.

A. *Finite Sequence Example*

Recall the former finite sequences example of Section II. This time, the correlation between $h[k] = [1, \hat{2}, 1, -1]$ and $x[k] = [\hat{1}, 2,3,1]$ is desired. The correlation sum is the summation:

$$r_{xh}[k] = \sum_{m=-\infty}^{\infty} x[m]h[m-k]$$

From the data $x[k]$ and $h[k]$, $-2 \le k \le 4$. And since $x[k]$ is causal the analytical solution is as follows:

$$r_{xh}[k] = \sum_{m=0}^{3} x[m]h[m-k]$$

$$r_{xh}[-2] = x[0]h[2] + x[1]h[3] = -1$$

$$r_{xh}[-1] = x[0]h[1] + x[1]h[2] + x[2]h[3] = 1 - 2 = -1$$

$$r_{xh}[0] = x[0]h[0] + x[1]h[1] + x[2]h[2] = 2 + 2 - 3 = 1$$

$$r_{xh}[1] = x[0]h[-1] + x[1]h[0] + x[2]h[1] + x[3]h[2] = 7$$

$$r_{xh}[2] = x[1]h[-1] + x[2]h[0] + x[3]h[1] = 2 + 6 + 1 = 9$$

$$r_{xh}[3] = x[2]h[-1] + x[3]h[0] = 3 + 2 = 5$$

$$r_{xh}[4] = x[3]h[-1] = 1$$

$$r_{xh}[k] = [-1, -1, \hat{1}, 7,9,5,1]$$

One can observe that analytical convolution and correlation seem to be of the same complexity, but arguably, it seems pretty more complex to determine the sequence $-2 \le k \le 4$ for correlation than the $-1 \le k \le 5$ for convolution (which is just obvious from looking at the two sequences to be convolved). To determine the sequence for correlation, first the two signals are juxtaposed at the $k = 0$ point.

k		-1	0	1	2	3	
$x[k]$			$\hat{1}$	2	3	1	
$h[k]$		1	$\hat{2}$	1	-1		

Then $x[0]$ and $h[2]$ are aligned to determine how much left shifting of $h[k]$ is required (advance). Two shifts were required as shown below, so k starts at -2.

k		-2	-1	0	1	2	3	
$x[k]$				$\hat{1}$	2	3	1	
$h[k]$	1	$\hat{2}$	1	-1				

Finally, $x[3]$ and $h[-1]$ are aligned to determine how much right shifting of $h[k]$ is required (delay). Four shifts were required so k ends at 4.

k		-2	-1	0	1	2	3	4		
$x[k]$				$\hat{1}$	2	3	1			
$h[k]$							1	$\hat{2}$	1	-1

Next, the vector multiplication approach is extended to correlation. The steps follow from the convolution solution, the signals to be correlated $x[k], h[k]$ are represented as row vectors. Then the correlation sum is defined as

$$r_{xy}[k] = \sum_{k=-\infty}^{\infty} \searrow [k] h^T x \qquad (5)$$

where $\sum_{k=-\infty}^{\infty} \searrow [k]$ indicates the *right diagonal* summations of the matrix obtained from $h^T x$. Let

$$r' = h^T x$$

Then

$$r' = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 1 \\ -1 & -2 & -3 & -1 \end{bmatrix}$$

Taking right diagonal sums of y' gives”

$$r_{xy}[k] = [-1, -1, \hat{1}, 7, 9, 5,1]$$

The tabularized vector multiplication solution is shown below:

$h[k]$ \ $x[k]$	$\hat{1}$	2	3	1	
1	1	2	3	1	
$\hat{2}$	2	4	6	2	1
1	1	2	3	1	5
-1	-1	-2	-3	-1	7
		-1	-1	1	$r_{xy}[k]$

Notice that the matrix obtained from vector multiplication is the same for both convolution and correlation. The difference in solution lies on which direction (left or right) the diagonal sums are taken. To determine the $k = 0$ point from this method it is still required to follow the procedure listed formerly to determine the k sequence for each given pair of signals.

IV. Circular (Periodic) Convolution

Circular (periodic) convolution is also an important concept in signals and systems. It can also be computed using the analytical and graphical approaches. In circular convolution, both sequences to be convolved are N-periodic and the convolution summation is over one period N. This differentiates it from the linear convolution summation which can be from $-\infty$ to $+\infty$. Examples of circular convolution using the graphical approach are presented in [1] (p. 349-351, 651-652), [3] (pp. 399-426), [4] (pp. 471-474), and [5] (pp. 676-687). However, the analytical and graphical solutions presented are usually more complex and involving than that of linear convolution.

To illustrate, consider Example 7.2.1 in [4] (p. 472) which is as follows. Perform the circular convolution

$$y[n] = x_1[n] \circledast x_2[n]$$

of the following two sequences:

$$x_1[n] = [\hat{2}, 1, 2, 1], x_2[n] = [\hat{1}, 2, 3, 4]$$

From the above data $N = 4$, and the analytical solution is obtained as follows:

$$y[n] = \sum_{k=0}^{3} x_1[k]x_2\left[((n-k))_N\right], 0 \le n \le 3$$

$$y[0] = x_1[0]x_2\left[((0))_4\right] + x_1[1]x_2\left[((-1))_4\right] + x_1[2]x_2\left[((-2))_4\right] + x_1[3]x_2\left[((-3))_4\right]$$

$$= 2 \times 1 + 1 \times 4 + 2 \times 3 + 1 \times 2 = 14$$

$$y[1] = 2x_2\left[((1))_4\right] + x_2\left[((0))_4\right] + 2x_2\left[((-1))_4\right] + x_2\left[((-2))_4\right]$$
$$= 2 \times 2 + 1 \times 1 + 2 \times 4 + 1 \times 3 = 16$$

$$y[2] = 2x_2\left[((2))_4\right] + x_2\left[((1))_4\right] + 2x_2\left[((0))_4\right] + x_2\left[((-1))_4\right]$$
$$= 2 \times 3 + 1 \times 2 + 2 \times 1 + 1 \times 4 = 14$$

$$y[3] = 2x_2\left[((3))_4\right] + x_2\left[((2))_4\right] + 2x_2\left[((1))_4\right] + x_2\left[((0))_4\right]$$
$$= 2 \times 4 + 1 \times 3 + 2 \times 2 + 1 \times 1 = 16$$

$$y[n] = \{\widehat{14}, 16,14,16\}$$

One may appreciate the added complexity of this solution in comparison to analytical linear convolution. In the above solution, even the part of determining the circular shifting $x_2\left[((n-k))_N\right]$, $0 \le n \le 3$ is not shown.

The graphical solution is presented in [4] (p. 473) using a series of circular discs to illustrate the concept of circular shifting. From observation of the series of circles in the solution in [4], the required circular shifting may be represented using one disc as shown below"

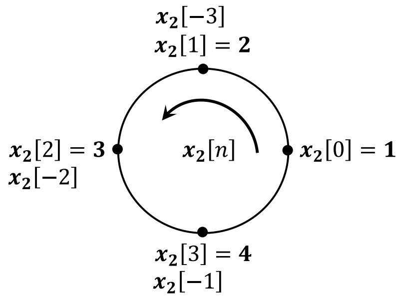

Fig. 2. Circular shifting requires one clockwise pass from $x_2[3]$ to $x_2[0]$ followed by another clockwise pass from $x_2[-1]$ to $x_2[-3]$. The sequence $x_2\left[((n-k))_N\right]$ is 4,3,2,1,4,3,2.

Let $x_2\left[((n-k))_N\right]$ be represented as $x_2^*[n]$ Then, the vector multiplication solution is presented as follows:

n	**3**	**2**	**1**	**0**	-1	-2	-3	
$x_2^*[n]$ / $x_1[n]$	4	3	2	1	4	3	2	
2	8	6	4	2	8	6	4	
1	4	3	2	1	4	3	2	
2	8	6	4	2	8	6	4	
1	4	3	2	1	4	3	2	**14**
					16	**14**	**16**	$y[n]$

$$y[n] = \{\widehat{14}, 16,14,16\}$$

In addition, it does not matter which direction the circular shifting is made. One can as well make one *anticlockwise* pass from $x_2[-3]$ to $x_2[0]$ followed by another anticlockwise pass from $x_2[1]$ to $x_2[3]$. In this case $x_1[n]$ is also taken anticlockwise. Then the vector multiplication is as follows"

n	-3	-2	-1	**0**	1	2	3	
$x_2^*[n]$ / $x_1[n]$	2	3	4	1	2	3	4	
1	2	3	4	1	2	3	4	
2	4	6	8	2	4	6	8	
1	2	3	4	1	2	3	4	
2	4	6	8	2	4	6	8	**16**
					14	**16**	**14**	$y[n]$

The solution is also taken in reverse as:

$$y[n] = \{\widehat{14}, 16,14,16\}$$

V. Commutativity Test of Convolution and Correlation

A main property that differentiates convolution from correlation is that convolution is commutative, i.e.

$$y[k] = x[k] * h[k] = x[k] * h[k]$$
$$\Rightarrow \sum_{m=-\infty}^{\infty} x[m]\, h[k-m] = \sum_{m=-\infty}^{\infty} h[m]\, x[k-m]$$

However, correlation is not, i.e.

$$r_{xy}[k] \ne r_{yx}[k]$$

The commutativity of convolution can be demonstrated by using the vector multiplication method presented here.

$$y[k] = \sum_{k=-\infty}^{\infty} \ltimes [k] x^T h = \sum_{k=-\infty}^{\infty} \ltimes [k] h^T x$$

Let

$$y_1' = h^T x, y_2' = x^T h$$

Then

$$y_2' = (y_1')^T$$

$$y_1' = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 1 \\ -1 & -2 & -3 & -1 \end{bmatrix}, y_2' = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 2 & 4 & 2 & -2 \\ 3 & 6 & 3 & -3 \\ 1 & 2 & 1 & -1 \end{bmatrix}$$

and

$$y[k] = y_1[k] = y_2[k] = [1, \hat{4}, 8, 8,3, -2, -1]$$

To show that correlation is not commutative:

$$r_{xy}[k] = \sum_{k=-\infty}^{\infty} \rtimes [k] h^T x$$

$$r_{yx}[k] = \sum_{k=-\infty}^{\infty} \rtimes [k] x^T h$$

Let

$$r_1' = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 1 \\ -1 & -2 & -3 & -1 \end{bmatrix}, r_2' = x^T h = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 2 & 4 & 2 & -2 \\ 3 & 6 & 3 & -3 \\ 1 & 2 & 1 & -1 \end{bmatrix}$$

Then

$$r_{xy}[k] = [-1, -1, \hat{1}, 7, 9, 5,1]$$

$$r_{yx}[k] = [1,5,9,7, \hat{1}, -1, -1] = r_{xy}[-k] \neq r_{xy}[k]$$

VI. Conclusion

The paper presented a unified vector multiplication with diagonal summation approach for calculating convolution and correlation of finite sequences. The paper is basically a theoretical analysis of the regular DT convolution.

Also, it is shown that for finite sequences, it does not require the Z-transform to reduce the convolution sum to a multiplication problem, rather, vector multiplication can be done directly without the Z-transform, thereby reducing complexity because it involves much fewer steps.

The approach presented here provides additional tools to adopt for practice. Future work may include investigating other simplified methods of solving convolution and correlation problems by hand. Investigating the computational complexities of the different approaches can also be considered.

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