Research on the Ultimate Velocity and the Ultimate Height of Gravity Center of Vehicles Going through Roadway Curve to Avert Rollover

Xi Cao, Naonori Yanagisawa, Naoki Kurosawa and Hideki Miyashita

Abstract — The results of a study on the ultimate velocity and the ultimate height of gravity center of vehicle to avert rollover while the vehicle going through roadway curve are reported in this paper. An improved mathematical model is developed. The relationship between the ultimate velocity to avert rollover and the relative parameters, and the variations of the ultimate height of gravity center of the vehicle to avert rollover with the relative parameters while the vehicle going through roadway curve are obtained. The results show that in order to avert rollover, the ultimate velocity for the bus involved in Karuizawa Bus Accident should be limited to 77.2 km/h while going through Karuizawa roadway curve. However, the bus velocity recorded was 96 km/h, which exceeded the ultimate velocity and can be considered as one of the important causes that lead to the bus rollover accident. The ultimate vehicle velocity decreases with wind speed increasing for the studied four types of vehicles. The larger the lateral project area of vehicle, the smaller the ultimate vehicle velocity to avert rollover. The ratio of $H/W$ (the ratio of the ultimate height of gravity center to distance between the right and left wheels) of vehicle will influence the ultimate vehicle velocity and the ultimate height of gravity center to avert rollover markedly. The longitudinal gradients $\theta$ of roadway surface plays a role in increasing the risk of vehicle rollover.

Keywords — curve, lateral project area, ratio of $H/W$, ultimate height of gravity center, ultimate velocity, wind speed.

I. INTRODUCTION

Many traffic accidents have occurred on roadway curves. Some accidents are due to losing control of the vehicles and then the vehicles quickly slipped out the road curves. Some vehicles failed to go through road curve but quickly went into a combined motion of sideslip and rollover after losing control of the vehicles on the road curve in the traffic accidents. In Japan, a typical and terrible roadway curve traffic accident called Karuizawa Bus Accident occurred on Jan.15, 2016, which resulted in 15 deaths including 13 young college students and 26 injured [1]. The driver of the tour bus lost control of the bus to go through a Karuizawa road curve and the bus skidded, overturned, rolled rightward and then tumbled down 3 meters from the road. This road curve accident revealed that the bus failing to go through the road curve was due to not only sideslip but also overturn motion. Fig. 1 shows a photo of Karuizawa Bus after the accident.

Therefore, in order to avert such accidents happening and reduce the numbers of traffic accident victims, it is important to investigate the factors that affect the safety of vehicles going through roadway curves.

In [1], the factors that lead to vehicles sideslip are analyzed and the ultimate velocities for vehicles going through roadway curves to avert sideslip are given under different conditions. In this paper, the results of the ultimate velocity and the ultimate height of gravity center of vehicles to avert overturn while the vehicles go through roadway curves are reported.

A previous model to calculate a vehicle overturn on road curve is given in [3]. In the model, the curve radius $R$ and road surface superelevation angle $\alpha$ are considered.

In this study, an improved mathematical model is established, in which besides road surface superelevation angle $\alpha$, the influences from the longitudinal gradient $\theta$ of the road curve surface and the wind force are also considered. The predicted results are given and discussed.

II. SYMBOLS

The meanings of parameters used in this paper are given as follows.

$A$ : vehicle lateral projection area, $m^2$
$C_D$ : wind force coefficient
$D$ : wind force, $N$
$e$ : atmosphere vapor pressure, $hPa$
$F_c$ : centrifugal force, $N$

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\[ g \text{ : acceleration of gravity, m/s}^2 \]
\[ H \text{ : height of gravity center, m} \]
\[ m \text{ : vehicle gross mass, kg} \]
\[ N \text{ : ground reaction force, N} \]
\[ n \text{ : seat number} \]
\[ p \text{ : atmosphere pressure, hPa} \]
\[ R \text{ : road curve radius, m} \]
\[ T \text{ : air temperature, °C} \]
\[ U \text{ : wind speed, m/s} \]
\[ V \text{ : velocity, km/h} \]
\[ v \text{ : velocity, m/s} \]
\[ W \text{ : distance between the right and left wheels, m} \]
\[ Z \text{ : height of center of vehicle lateral projection area, m} \]
\[ \alpha \text{ : road surface superelevation angle, °} \]
\[ \theta \text{ : road surface longitudinal gradient, °} \]
\[ \rho \text{ : density of air, kg/m}^3 \]

### III. MATHEMATICAL MODEL FOR A VEHICLE ROLOVER MOTION

Reference [3] gives a mathematical model to analyze a vehicle overturn on a road curve, in which, the influence of the surface superelevation angle \( \alpha \) on vehicle overturn is taken into consideration. From the analysis based on this model, it can be expressed as that in order to keep a vehicle from overturn, its ultimate velocity must satisfy the condition given by formula below:

\[
v \leq \sqrt{\frac{gR}{2H \sin \alpha + \frac{W \cos \alpha}{2H \cos \alpha - W \sin \alpha}}}
\]  

(1)

Because in many cases, the traffic accidents often happened on the road curves whose surfaces are not only with superelevation angle \( \alpha \) but also with the longitudinal gradient \( \theta \), we established an improved mathematical model, in which the effects of the longitudinal gradient \( \theta \) of road curve surface and wind force on vehicle rollover motion are also taken into consideration.

First of all, the model in which the factors of superelevation angle \( \alpha \) and the longitudinal gradient \( \theta \) are considered is built. In the analysis, it is assumed that the studied object, a vehicle at a constant velocity is going through a road curve whose radius is \( R \). A non-inertial spatial coordinate system (\( x, y, z \)) is fixed on the mass point of the vehicle as shown in Fig. 2 and Fig. 3. The direction of the vehicle traveling is shown in the figures.

![Fig. 2. Top view of the vehicle from above (\( \alpha \) and \( \theta \) considered), (x-y plane)](image)

The mechanical drawing for analyzing the rollover motion of a vehicle in \( y-z \) plane is shown in Fig. 4.

![Fig. 3. Lateral view from the right side of the vehicle (\( \alpha \) and \( \theta \) considered), (y-z plane)](image)

In Fig. 2~Fig. 4, \( F_c \) is the centrifugal force acting on the vehicle and could be expressed as:

\[
F_c = m \frac{v^2}{R}
\]  

(2)

If a vehicle overturns and rolls rightward, its left wheel will leave the road (ground) and the vehicle body will roll rightward. In other words, the rollover motion of the vehicle body is a rolling motion around right wheel support point \( O_2 \) that is the contact point of the right wheel and road surface. In \( x-z \) plane, the resultant moment of forces around right wheel support point \( O_2 \) is the cause and motive power to result in the vehicle overturn.

Therefore, the analysis starts from establishing the equation of the resultant moment of forces around support point \( O_2 \) in \( x-z \) plane.

Equation of equilibrium of resultant moment of forces around the contact point of outside wheel and road surface is given as followings:

\[
N_1 W - mg \sin \alpha - mg \cos \theta \cos \frac{w}{2} + F_c \cos \alpha H - F_c \sin \alpha \cos \frac{w}{2} = 0
\]  

(3)

\[
N_1 W + F_c (H \cos \alpha - \frac{w}{2} \sin \alpha \cos \theta) - mg (H \sin \alpha + \frac{w}{2} \cos \alpha \cos \theta) = 0
\]  

(4)

Substituting \( F_c = m \frac{v^2}{R} \) into formula (4):

\[
N_1 W + m \frac{v^2}{R} (H \cos \alpha - \frac{w}{2} \sin \alpha \cos \theta) - mg (H \sin \alpha + \frac{w}{2} \cos \alpha \cos \theta) = 0
\]  

(5)
\[ N_1 = m \frac{v^2}{R} \left( \frac{1}{2} \sin \theta - \frac{H}{W} \cos \alpha \right) + mg \left( \frac{H}{W} \sin \alpha + \frac{1}{2} \cos \alpha \cos \theta \right) \]

As long as the vehicle does not turn over rightward, the reaction force acted on the inside wheel, \( N_1 \), will satisfy the condition:

\[ N_1 \geq 0 \tag{7} \]

By substituting formula (6) into inequality (7), the following inequality is obtained:

\[ m \frac{v^2}{R} \left( \frac{1}{2} \sin \theta - \frac{H}{W} \cos \alpha \right) + mg \left( \frac{H}{W} \sin \alpha + \frac{1}{2} \cos \alpha \cos \theta \right) \geq 0 \tag{8} \]

From the inequality (8), the ultimate velocity for the vehicle going through roadway curves to avert overturn can be acquired as followings:

\[ v \leq \sqrt{\frac{gR}{2H(\sin \theta + W \cos \alpha)}} \tag{9} \]

This formula indicates that as a vehicle goes through a curve, its radius being \( R \), the height of gravity center being \( H \), the distance between the right and left wheels being \( W \), road surface superelevation angle being \( \alpha \) and longitudinal gradient being \( \theta \), in order to avoid rollover, the ultimate vehicle velocity \( v \) must satisfy the condition given by (9).

Next, to investigate the influence induced by wind force on the ultimate velocity of a vehicle driven on a road curve to avert rollover, the mechanical model shown in Fig. 5 is considered. In addition to the same conditions mentioned above, it is assumed that wind-induced lateral force \( D \) acts vertically on the side surface of the vehicle, and directs outward from the center of the curve in the \( x \) axial direction as shown in Fig. 5.

![Fig. 5. Back view of the vehicle (\( \alpha, \theta \) and \( D \) considered), (x-z plane).](image)

Equation of equilibrium of resultant moment of forces around the contact point of outside wheel and road surface, \( O_z \), is given as follows:

\[ N_1 W - mg \sin \theta - H - mg \cos \theta \cos \frac{W}{2} + Fc \cos \theta H - Fc \sin \theta \frac{W}{2} + DZ = 0 \tag{10} \]

\[ N_1 W + Fc (H \cos \frac{W}{2} \sin \theta) - mg (H \sin \frac{W}{2} \cos \theta) + DZ = 0 \tag{11} \]

Substituting \( Fc = m \frac{v^2}{R} \) into formula (11):

\[ N_1 W + m \frac{v^2}{R} (H \cos \frac{W}{2} \sin \theta) - mg (H \sin \frac{W}{2} \cos \theta) + DZ = 0 \tag{12} \]

As long as the vehicle does not turn over rightward, the reaction force acted on the inside wheel \( N_1 \) will satisfy the condition below:

\[ N_1 \geq 0 \]

By substituting formula (13) into inequality (7), the following inequality is obtained:

\[ m \frac{v^2}{R} (\frac{1}{2} \sin \theta - \frac{H}{W} \cos \alpha) + mg ((\frac{H}{W} \sin \alpha + \frac{1}{2} \cos \alpha \cos \theta) - \frac{DZ}{W} \geq 0 \tag{14} \]

\[ m \frac{v^2}{R} \left( \frac{W \sin \theta \cos \alpha}{2W} \right) + mg \left( \frac{2H \sin \theta + W \cos \alpha}{2W} \right) - \frac{DZ}{W} \geq 0 \tag{15} \]

Multiplying both sides of formula (15) by 2WR gives:

\[ mv^2 \left( \frac{W \sin \theta \cos \alpha}{2W} \right) + R mg \left( 2H \sin \theta + W \cos \alpha \right) - 2RDZ \geq 0 \tag{16} \]

\[ R mg \left( 2H \sin \theta + W \cos \alpha \right) - 2RDZ \geq mv^2 \left( 2H \sin \theta + W \cos \alpha \right) \tag{17} \]

From the inequality (17), the ultimate velocity for the vehicle going through roadway curves and acted up by wind force to avert rollover can be acquired as followings:

\[ v \leq \sqrt{\frac{R mg (2H \sin \theta + W \cos \alpha) - 2RDZ}{2H(\sin \theta + W \cos \alpha)}} \tag{18} \]

Substituting \( D = C_D \frac{1}{2} U^2 A \) [4] into formula (18):

\[ v \leq \sqrt{\frac{R mg (2H \sin \theta + W \cos \alpha) - C_D \rho U^2 AZ}{2H(\sin \theta + W \cos \alpha)}} \tag{19} \]

This formula indicates that when a vehicle acted up by wind force goes through a roadway curve, its radius being \( R \), the height of gravity center being \( H \), the distance between the right and left wheels being \( W \), road surface superelevation angle being \( \alpha \) and longitudinal gradient being \( \theta \), in order to avoid rollover, the ultimate vehicle velocity \( v \) must satisfy the condition given by formula (19).

The height of gravity center of vehicle may be an important factor to affect vehicle rollover. Therefore, the relationship
between the ultimate height of gravity center of vehicle going through roadway curves to avert rollover and relative parameters is derived as below.

Spreading out the inequality (16):

\[
m v^2 W \sin \alpha \cos \theta - H (2 m v^2 \cos \alpha) + R W mg \cos \alpha \cos \theta - 2 R D Z \geq 0 \tag{20}
\]

\[
H (2 R m g \sin \alpha - 2 m v^2 \cos \alpha) + W m \cos \theta (v^2 \sin \alpha + g R \cos \alpha) - 2 R D Z \geq 0 \tag{21}
\]

From the inequality (21), the ultimate height of gravity center of vehicles to avert rollover can be acquired as follows:

\[
H \leq \frac{W m \cos \theta (v^2 \sin \alpha + g R \cos \alpha) - 2 R D Z}{2 m (v^2 \cos \alpha - g R \sin \alpha)} \tag{22}
\]

Substituting \(D = C_D \rho U^2 A\) into formula (22):

\[
H \leq \frac{W m \cos \theta (v^2 \sin \alpha + g R \cos \alpha) - C_D \rho U^2 A R Z}{2 m (v^2 \cos \alpha - g R \sin \alpha)} \tag{23}
\]

This formula indicates that when a vehicle acted up by wind force goes through a roadway curve at a velocity of \(v\), its radius being \(R\), the distance between the right and left wheels being \(W\), road surface superelevation angle being \(\alpha\) and longitudinal gradient being \(\theta\), in order to avoid rollover, the ultimate height of gravity center \(H\) must satisfy the condition given by formula (23).

IV. DETERMINING THE PARAMETERS USED IN THE PRESENT RESEARCH

A. The Studied Road Curve Sections

The studied road models are two road curve sections respectively on two existing roadways in Nagano Prefecture, Japan. One is the road curve section where serious Karuizawa Bus Accident has occurred. The detailed introduction of the road curve section is given in [1] and omitted here. Another is Sugadaira curve section on Japan National Route 406. Some photos of the Sugadaira curve section are shown in Fig. 6, 7.

The accurate spot locations and shapes of the road curve sections studied in the investigation is shown respectively in Fig. 8 and Fig. 9 with the red cross on the cut maps.

Fig. 6. The view of the Sugadaira road curve [5].

Fig. 7. Southern side view of the Sugadaira road curve [5].

Fig. 8. The shape of Karuizawa Bus Accident spot road curve (with a latitude of 36°19’36.7″N and a longitude of 138°38’43.7″E) [1], [5].

Fig. 9. The shape of the Sugadaira road curve (with a latitude of 36°33’07.2″N and a longitude of 138°19’59.1″E) [5].
B. The Method to Determine Road Parameters and the Measured Results

The method to determine the road parameters including curve radius $R$, angle of road surface superelevation $\alpha$, road surface longitudinal gradient $\theta$ is illustrated in [1]. Its explanation is omitted here.

The radius of Sugadaira road curve is obtained and the method is graphically illustrated in Fig.10 and the detail description is given in [1].

![Fig. 10. The Sugadaira road curve $R$ [1], [5].](image1)

The road surface superelevation angle $\alpha$ and longitudinal gradient $\theta$ are determined by means of actually measuring. Fig. 11 illustrates the road surface superelevation angle $\alpha$, and Fig. 12 shows the scene of actually measuring angle $\alpha$. The measured part results are summed up in Table I.

![Fig. 11. The illustration of the road surface superelevation angle $\alpha$ (the back view).](image2)

![Fig. 12. The scene of actually measuring angle $\alpha$ [1], [6].](image3)

<table>
<thead>
<tr>
<th>Curve</th>
<th>Karuizawa road curve</th>
<th>Sugadaira road curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (m)</td>
<td>80</td>
<td>21</td>
</tr>
<tr>
<td>$\alpha$ (°)</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$\theta$ (°)</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

C. Choosing the Studied Vehicles

In the study, four types of vehicles, bus, truck, standard-sized car and light car, are adopted as the studied objects. A tour bus is chosen as one of the studied objects that is high decker equipped with two row seats salon, Hino S'elega RU1ESBA [7], which is quite similar to the bus involved in Karuizawa Bus Accident. The chosen truck is a production manufactured by Mitsubishi Fuso Truck & Bus Company, Corp., the model of which is APG-FU64VTZ3XSV [8]. The model of the standard-sized car is SUBARU HYBRID 2.0i EyeSlight [9]. The light car model adopted is Town Box G Special 2WD manufactured by Japan’s Mitsubishi Motors Company [10]. The part appearance diagrams of the studied vehicles together with determination of vehicle parameters will be shown in next section.

D. The Method to Determine the Vehicle Parameters

The vehicle parameters include the lateral projection area of vehicle $A$ $(m^2)$, the distance between the right and left wheels $W$ (m), the vehicle gross mass $m$ (kg), the wind force coefficient $C_d$, the height of gravity center of vehicle $H$ (m), and the height of center of vehicle lateral projection area $Z$ (m).

1) Determining the lateral projection area ($A$) and the distances between the right and left wheels ($W$) of the vehicles

The two parameters are measured and calculated by means of a software called 10-0! Excel for the Measuring of Length and Area [11]. Fig.13 shows how to use the software to measure and calculate the parameters. As long as the drawing of the studied object vehicle is input to the Excel sheet of the software, and the range of the area to be measured is designated with yellow line, the calculated area $A$ will appear as shown by red underline in Fig. 13. Fig. 13 shows the measuring and calculating the lateral projection area $A$ of the studied tour bus [7], [11], in which the screen displays the value of the lateral projection area $A$ calculated is 392173.74 cm$^2$ with red underline ($= 39.217$ m²). The bus drawing is obtained from the Web HP of Hino Auto Body, Ltd, and can be considered as an accurate drawing.

The distances between the right and left wheels $W$ can be obtained similarly, but the difference is by designating the line connecting two-wheel tire centers contacting road surface in yellow. The wheel distance $W$ for the studied truck [8] is measured as 183.3 cm ($=1.833$ m). The truck back view drawing is obtained from the Web Home Page set up by Mitsubishi-Fuso Truck and Bus Corp., and can be considered as an accurate drawing.

The measured and calculated parameters $A$ and $W$ of other types of vehicles are summed up in Table II. For the case that the distance between two front wheels is different from that between two back wheels, the short one is adopted.
2) **Determining the vehicle gross mass (m)**

The vehicle gross mass is the sum of the vehicle body mass and the total mass of riding capacity, counting 55 kg per person. For truck, besides the mass mentioned, the maximum loading weight (14,100 kg) is added to the gross mass. The relative data are picked up from their specification sheets displayed respectively on the web Home Page set up by their manufacturers [8]-[10]. Since the data on Hino S’elega RU1ESBA was not found, instead, the data on Hino S’elega HD [12] that is of the same stripe with Hino S’elega RU1ESBA are used. The values of the calculated gross masses of the vehicles are summed up in Table II.

3) **Determining the wind force coefficient (C_D)**

The values of the wind force coefficient $C_D$ depend on the configuration of the vehicle. The coefficients of four types of vehicles are estimated depending on their configuration and referencing the examples of the wind force coefficients for different configurations of bodies provided in [13]. The illustration of the obtained value of the coefficient $C_D$ of wind-induced lateral force acting on the studied light car is shown in Fig. 14. The values of the coefficients for other vehicles are summed up in Table II.

4) **Determining the height of center of vehicle lateral projection area (Z)**

It is considered that wind force acts at center of vehicle lateral projection area in the direction outward from the curve’s center. In the investigation into the vehicle, only the height of center of vehicle lateral projection area, Z, need to be determined as shown in Fig. 5. The method is illustrated in Fig. 15 and Fig. 16, taking the studied bus as the measured object. The uniformity paper model of the lateral view figure of the bus is obtained by cutting its accurate and scaled-down figure from [7]. We hanged the paper model in three different directions and found the center of the figure as illustrated in Fig. 15 and Fig. 16. The introduction of the measuring the center of the figures of other three types of vehicles is omitted. The measured results are summed up in Table II.

5) **Determining the height of gravity center of vehicle (H)**

The height of gravity center of vehicle is affected by cargo and rate of its riding capacity. It is assumed that the 4/5 height of center of vehicle lateral projection area is the height of gravity center. Fig. 17 shows the value of $H$ of the standard-sized car adopted in the studied. The values of $H$ of other studied vehicles are given in Table II.
By summing up the determined, measured, and calculated results above, the values of the vehicle parameters are summed up in Table II, in which $n$ stands for the number of seats including drivers’.

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Bus</th>
<th>Truck</th>
<th>Standard-sized Car</th>
<th>Light Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (m$^2$)</td>
<td>39.217</td>
<td>40.071</td>
<td>4.886</td>
<td>5.256</td>
</tr>
<tr>
<td>Vehicle mass (kg)</td>
<td>12290</td>
<td>10755</td>
<td>1510</td>
<td>970</td>
</tr>
<tr>
<td>$n$</td>
<td>47</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$m$ (kg)</td>
<td>14875</td>
<td>24965</td>
<td>1785</td>
<td>1190</td>
</tr>
<tr>
<td>$C_D$</td>
<td>1.10</td>
<td>1.20</td>
<td>0.85</td>
<td>1.05</td>
</tr>
<tr>
<td>$W$ (m)</td>
<td>1.820</td>
<td>1.833</td>
<td>1.535</td>
<td>1.280</td>
</tr>
<tr>
<td>$Z$ (m)</td>
<td>1.928</td>
<td>1.874</td>
<td>0.781</td>
<td>0.966</td>
</tr>
<tr>
<td>$H$ (m)</td>
<td>1.542</td>
<td>1.499</td>
<td>0.625</td>
<td>0.773</td>
</tr>
<tr>
<td>$H/W$</td>
<td>0.847</td>
<td>0.818</td>
<td>0.407</td>
<td>0.604</td>
</tr>
</tbody>
</table>

TABLE II: THE VEHICLE PARAMETERS

6) Meteorological data used in the analysis

Meteorological data used in the analysis are chosen from the actually observed weather data [14] on Jan. 15, 2016, when the Karuizawa Bus Accident occurred. The observed time is 2 o’clock when the observed atmospheric pressure $p$ is 895.9 hPa, the air temperature $T$ is -3 °C and the atmosphere vapor pressure $e$ is 3.1 hPa. Substituting the three parameters into the following formula [15]:

$$\rho = \frac{1.293}{1+0.003677} \cdot \frac{p}{1013} \cdot \left(1 - \frac{0.378e}{p}\right)$$ (24)

Air density $\rho = 1.1548 \text{kg/m}^3$ is obtained. The actually observed wind velocity $U = 2.9 \text{ m/s}$.

V. RESULTS AND DISCUSSION

Using the developed mathematical model expressed by the inequality (19), (23) etc., the relationships between the ultimate velocity or the ultimate height of gravity center of vehicles going through roadway curves to avert rollover and other parameters are obtained. In this and next section, the ultimate vehicle velocity and the ultimate height of gravity center of vehicle to avert rollover as the vehicle goes through roadway curve are abbreviated respectively to the ultimate velocity and the ultimate height of gravity center.

Using Inequality (19), the ultimate velocities with the unit of km/h for four types of vehicles travelling on the studied two road curves respectively under the weather condition in which the Karuizawa Bus Accident occurred are calculated, and the results are given in Table III.

From the results, in order to avert rollover, the ultimate velocity for the studied bus going through Karuizawa roadway curve should be limited to 77.2 km/h. However, on the day when the Karuizawa Bus Accident occurred the bus velocity recorded was 96 km/h, which exceeded the ultimate velocity and can be considered as one of the important causes that lead to the bus accident, which explains why the bus turned over and rolled rightward.

The ultimate velocity for the same bus going through Sugadaira road curve is much lower than that as going through Karuizawa road curve. The huge difference between them should be attributed to the difference of their radius. The radius of Sugadaira road curve is much smaller than the radius of Karuizawa road curve as shown in Tab.1. The smaller the radius $R$, the lower the ultimate velocity as expressed by inequality (19).

The difference of ultimate velocity between different types of vehicles can be attributed to the difference between the heights of gravity center of the vehicles $(H)$, or the ratio of $H/W$ of the vehicles, $W$ being the distance between the right and left wheels of vehicle. Comparing the values of $H/W$ given in Table II and the ultimate velocities given in Tab.3, it can be observed that their magnitude orders are corresponding each other. The coincidence can be further confirmed by Fig.18, in which the continuous graphs for the vehicles going through Sugadaira road curve $(\theta=3^\circ$ and $\alpha=4^\circ$) varying with radius are shown.

The relations between the ultimate velocity and wind speed for four types of vehicles going through the Karuizawa road curve are shown in Fig. 19. The air density is calculated by formula (24) using the observed weather data on the day when the Karuizawa bus accident occurred, and $\rho = 1.1548 \text{kg/m}^3$. With wind speed increasing, the ultimate velocities for all types of vehicles decrease, in which light car drops most quickly. The similar trend has been observed in the analysis of the relationship between the ultimate velocity

Fig. 17. The calculated height of gravity center of the studied standard-size car.

Fig. 18. The variation of the ultimate vehicle velocity with curve radius for four types of vehicles.

TABLE III: THE ULTIMATE VELOCITY (km/h) TO AVERT ROLLOVER FOR FOUR TYPES OF VEHICLES

<table>
<thead>
<tr>
<th>Vehicle Road curve</th>
<th>Bus</th>
<th>Truck</th>
<th>Standard-sized Car</th>
<th>Light Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karuizawa</td>
<td>77.2</td>
<td>78.6</td>
<td>111.5</td>
<td>91.4</td>
</tr>
<tr>
<td>Sugadaira</td>
<td>42.8</td>
<td>43.5</td>
<td>61.5</td>
<td>50.3</td>
</tr>
</tbody>
</table>
to avert sideslip and wind speed as shown in Fig. 20 [1]. This can be understood as that a light object is relatively more easily affected by wind force in general. In the analysis of vehicle sideslip motion, the ultimate velocity variation with the wind speed for bus is not the lowest one among the four types of vehicles but here the bus ultimate velocity is the lowest in the range of wind speed varies from 0 m/s to 30 m/s. That is because the ratio of $H/W$ for the bus is the highest in four types of vehicles, which affects the rollover obviously.

Fig. 19. The relations between the ultimate velocity and wind speed for four types of vehicles going through Karuizawa road curve to avert rollover.

Fig. 20. The relationships between the ultimate velocity and wind speed for four types of vehicles to avert sideslip [1].

In Fig. 21 the influence of the ratio of $H/W$ on rollover can be more obviously seen. With the road curve radius enlarging the ultimate velocity increases for all cases of ratio of $H/W$. If the road curve radius is the same, the bigger the ratio of $H/W$, the smaller its ultimate velocity.

Fig. 21. The relationships between the ultimate vehicle velocity and road curve radius for different ratios of $H/W$ (truck, $\theta = 4^\circ$, $\alpha = 0^\circ$).

Fig. 22. The variation of the ultimate vehicle velocity with wind speed for different values of the lateral projection area of vehicle $A$.

The relations between the ultimate velocity for the studied bus going through road curves whose longitudinal gradients $\theta$ is the same as that of the Karuizawa road curve, $\theta = 4^\circ$, and the curve radius for different values of superelevation angle $\alpha$ of roadway surface are shown in Fig. 23. It is clear that the ultimate velocities increase with the increase of the road curve radius $R$ for all superelevation angle $\alpha$. For the same curve radius, the bigger the superelevation angles $\alpha$, the higher the ultimate velocity, which gives the fact that the superelevation angle $\alpha$ plays a role in reducing the risk of vehicle rollover.

Fig. 23. The variation of the ultimate vehicle velocity with curve radius for different superelevation angles.
The relations between the ultimate velocity for the studied truck going through road curves whose superelevation angle $\alpha$ is the same as that of the Karuizawa road curve, $\alpha=0^\circ$, and wind speed for different longitudinal gradients $\theta$ of roadway surface are shown in Fig. 24. Other conditions are the same as above. Watching the variation of the ultimate velocity with the wind speed, it can be seen that with the increase of wind speed, the ultimate vehicle velocities for different longitudinal gradients go down. For the same wind speed, the bigger the longitudinal gradients $\theta$, the lower the ultimate velocity, which explains that longitudinal gradients $\theta$ plays a role in increasing the risk of vehicle rollover.

![Fig. 24. The variation of the ultimate vehicle velocity with wind speed for different longitudinal gradients.](image)

In Fig. 25 the relationships between the ratios of $H/W$ of the studied bus and vehicle velocity to avert rollover while the studied bus going through road curves whose superelevation angle $\alpha$ and the longitudinal gradients $\theta$ are the same as those of the Sugadaira road curve, $\alpha=4^\circ$, and $\theta=3^\circ$, for different road curve radiiues are given. It can be observed that with the increase of the vehicle velocity, the ultimate height of gravity center (expressed in ratio of $H/W$) drops for all curve radius cases. In order to satisfy the same ultimate height of gravity center or ratio of $H/W$, the smaller the curve radius, the lower the velocity must be for the bus going through the curve. It is estimated that the similar relationship can be obtained for other types of vehicles.

![Fig. 25. The relationships between the ratio of the ultimate height of gravity center to the distance between the right and left wheels of the studied bus and velocity for different road curve radiiues.](image)

In Fig. 26 the relationships between the ultimate height of gravity center (expressed in $H/W$) and the wind speed for four types of vehicles going through the Karuizawa road curve at velocity of 88 km/h are shown. For all types of vehicles, the ratio $H/W$ drops with the wind speed increasing. The order in quickly dropping for vehicles is light car, bus, truck, and standard-sized car.

![Fig. 26. The relationships between the ultimate height of gravity center (expressed in $H/W$) of vehicles to avert rollover and wind speed for four types of vehicles.](image)

It should be mentioned again $[1]$ that in this study the vehicle velocity is supposed unchangeable, i.e., vehicle’s acceleration is not taken into consideration. But in actuality as the component of the resultant force in any direction is not in a state of equilibrium, the acceleration in that direction will occur, which will be further investigated in our following studies.

**VI. CONCLUSIONS**

The improved mathematical model is built to analyze the factors affecting the ultimate velocity and the ultimate height of gravity center to avert rollover for vehicles going through road curves. Four types of vehicles (bus, truck, standard-size car and light car) are used as the studied objects, and the two really existing road curve sections located in Nagano prefecture are adopted as road models, one of which is the spot where serious Karuizawa Bus Accident has happened. The factors affecting the traffic safety for vehicle going through road curve are investigated from different aspects, and the results are obtained. We hope the results will provide the references for driving safely and traffic safety management to reduce the occurrence rate of vehicle rollover accident.

Through analyzing the results obtained, the following conclusions can be drawn.

1. For different types of vehicles, different shapes of road curves, in different weather conditions, the ultimate vehicle velocity and the ultimate height of gravity center to avert vehicle rollover vary. To avert rollover accident, the drivers and the managers of the traffic companies should know the ultimate velocity and ultimate height of gravity center.

2. In order to avert rollover, the ultimate velocity for the
The bus involved in Karuizawa Bus Accident should be limited to 77.2 km/h while going through Karuizawa roadway curve in the weather condition on the accident day. However, the bus velocity recorded was 96 km/h, which exceeded the ultimate velocity and can be considered as one of the important causes that lead to the bus rollover accident.

3. The ultimate vehicle velocity decreases with wind speed increasing for the studied four types of vehicles. The larger the lateral projection area of vehicle, the smaller the ultimate vehicle velocity to avert rollover.

4. Increasing the curve radius could increase the ultimate vehicle velocity and the ultimate height of gravity center to avert rollover.

5. The ratio of $H/W$ of vehicles will influence the ultimate vehicle velocity and the ultimate height of gravity center to avert rollover markedly. The smaller the value of $H/W$, the larger the ultimate vehicle velocity and the ultimate height of gravity center to avert rollover.

6. The longitudinal gradients $\theta$ of roadway surface plays a role in increasing the risk of vehicle rollover. However, the superelevation angle $\alpha$ of roadway surface plays a role in reducing the risk of vehicle overturn.

REFERENCES


[11] $\sigma_0$ 0! Excel for the Measuring of Length and Area.


[14] Japan Meteorological Agency (JMA), The past weather data.