Thermodynamic – Informational Paradigm in the Context of the Formation of a Mathematical Model of Transient Processes in an Open System

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Abstract—In this article the theoretical substantiation on the developed mathematical model of the analysis of transient processes of interacting heterogeneous hyperchaotic systems of fractional order is given. The model represents a thermodynamically spectrum of the dimensions of Poincare's return time. The thermodynamic category in open system is the entropy of Tsallis in fractional space.

Index Terms—Transient Process; Heterogeneous Hyperchaotic System; Thermodynamic Category; Poincare; Tsallis Entropy.

I. INTRODUCTION

Thermodynamic is a field of knowledge allowing one to consider the processes of transformation of energy and matter in different systems from a unified position. It is appropriate to note that the basis of dissipative structures and synergetic lies in the laws of thermodynamic.

The development of the information-theoretical approach to the modeling of self-organization processes in open systems catalyzed the possibility of combining the dynamic and information components in the implementation of various tasks.

In the conditions of competition of information and physical processes, evolutionary development of complex systems begins to be determined by its information properties, including their relation to the external environment [1]-[4].

Such systems, along with the usual exchange of energy and negentropy with the external environment, receive an additional possibility of their complication.

Thus, for example, when supercritical values of control parameters are reached in self-organizing open systems, forced transitions occur between stationary states.

In the case, dissipative structures arise, which are conserved only when there is pumping of energy from the surrounding medium [4].

Thus, when modeling self-organization processes in open systems that are in disequilibrium with the environment, it is necessary to take into account both the physical exchange processes between the open system and environment, and the exchange of information flow [4].

In this connection it is appropriate to note that the basis of dissipative structures and synergetic lies in the laws of thermodynamic.

If Prigogine extended the theory of dissipative structures to physico-chemical processes, his followers replaced the concept of the "exchange of thermal energy" with the definition of "exchange of matter", the "exchange of information" and thereby universalized the thermodynamic categories.

In the context of strengthening the position of the universalization of thermodynamic categories, one should pay attention to the work where exchange between temperature and entropy of Tsallis as Clausius entropy is considered.

Remark:

The complex motion in a nonlinear dissipative system is described in two ways: either by the kinetic theory methods (the thermodynamics or irreversible nonlinear processes), or by the methods of the Poincare theory developed initially for Hamiltonian systems. The rapid development in statistical theory of open systems and the theory of self-organization led to the synthesis of these two scientific areas.

II. BASIC PROVISIONS

On the central ideas in thermodynamics is the concept of entropy, which measures the state of disorder in the system.

The change in the state of entropy, upon transition from one thermodynamic state to another, qualifies entropy as a quasi-statistical parameter.

We consider the thermodynamic category in open systems as the entropy of Tsallis.

Exchange between interacting non-extensive systems of heterogeneous types with an open system will most correctly contribute to the formation of a mathematical model of transient processes.

It should be noted that the nontriviality of the studies is due to the use of hyperchaotic maps of fractional order with generalized memory.

Before proceeding to the realization of the task in hand, we will present a material characterizing the main parameters of the interacting systems in an open system.

A. Stability

Let us consider the results of investigations of the stability properties of the new entropy according to the proposed stability criteria for Lesche [4].

Lesche stability

In [5] Lesche proposed a stability criterion to study the stability of the Renyi entropy function.

Published on October 31, 2017.

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Let p and p' be two probability assignments, Lesche stability requires that $\forall \varepsilon > 0$ we can find a $\delta > 0$ such that:

$$\sum_{j=1}^n |p_j - p'_j| \leq \delta \Rightarrow \frac{|S_q[p'] - S_q[p]|}{S_q^{\max}} < \varepsilon. \quad (1)$$

The authors of the study [4] and [5], using generalized entropy transformations, formulated conditions that simplify the solution of the problem of Lesche stability. In [5] it is shown that Lesche stability is satisfied if:

$$\frac{|S_q[p'] - S_q[p]|}{S_q^{\max}} < C \sum_{j=1}^n |p_j - p'_j|, \quad (2)$$

where the constant C is given by:

$$C = \frac{f^{-1}(0^+) - f^{-1}(1^-)}{f^{-1}(0^+) - \int_0^1 f^{-1}(t) dt}, \quad (3)$$

and the function $f^{-1}(t)$ is the inverse of the probability distribution [4]. Therefore if $C \neq 0$, we can take an arbitrary δ as $\delta = \varepsilon / C$ such that the criterion in (1) is fulfilled. Then we see that in our case:

$$f^{-1}(t) = (-\ln t)^q - q(-\ln t)^{q-1}, \quad (4)$$

such that replacing in (3) we obtain:

$$C = \lim_{t \rightarrow 0^+} \frac{f^{-1}(t)}{f^{-1}(t) - \Gamma(1+q) + q\Gamma(q)} = 1, \quad (5)$$

and taking $\delta = \varepsilon$ the inequality in (2) is satisfied and thus (1).

B. Thermodynamic Stability

As is known, in the Boltzmann-Gibbs formalism the concavity of the entropy, $\frac{\partial^2 S}{\partial U^2} < 0$, is equivalent to the condition of thermodynamic stability. It has been recently shown [4], [5] that for the case of a non-additive entropy the property of concavity does not imply thermodynamic stability. The study of thermodynamic stability of some non-additive entropies can be found in [4], [5]. Given an isolated system composed of two independent and identical subsystems in equilibrium in which there is a small amount of internal energy transferred from one to the other, the property $\frac{\partial^2 S_q}{\partial U^2} < 0$ does not necessary implies that

$$S_q(U, U) > S_q(U + \Delta U, U - \Delta U), \quad (6)$$

provided that the initial state is a state that maximizes the total entropy. After some algebra we find

$$0 > \frac{1}{2} S_q'' + \sum_{k=1}^{\infty} \binom{q}{k} \left(\frac{1}{2} (S_{q-k}'' S_k'' + S_{q-k}'' S_k') - S_k' S_{q-k}' \right). \quad (7)$$

In the micro canonical picture $S_q = \ln^q n$, and we can analyze the summation in (7) by writing the functions S_k' and S_k'' in terms of S_k as follows

$$\begin{aligned} S_k' &= \frac{k}{n} S_{k-1} \\ S_k'' &= -\frac{k}{n^2} (S_{k-1} - (k-1) S_{k-2}) \\ S_k S_{q-k} &= S_q, \end{aligned} \quad (8)$$

such that replacing these identities into (7) and performing the summations we find

$$0 > \frac{1}{2} S_q'' - \frac{2^q - 1}{2n^2} (q S_{q-1} + (q - q^2) S_{q-2}), \quad (9)$$

which due to the fact that S_q is a positive and concave function (9) satisfies the inequality for $0 < q < 1$ and therefore thermodynamic stability. A simple check shows that the condition $0 > S''(q)$ is recovered for $q = 1$.

C. The Fractional Entropy

The Tsallis fractional entropy [5] is formulated by:

$$T_\lambda(\wp) = \frac{\int_\Lambda [\wp(\Lambda)]^\lambda d\Lambda - 1}{1 - \lambda}, \quad \lambda \neq 1, \quad (10)$$

or in discrete form:

$$T_\lambda(\wp) = \frac{1}{\lambda - 1} \left(1 - \sum_{k=1}^m \wp_k^\lambda \right), \quad \lambda \neq 1. \quad (11)$$

On this level, we require a computation of an applicable aggregate of the information created by noticing the entrance of an occurrence having probability $\wp \in [0, 1]$. In this discussion, we suggest the functional entropy formula:

$$T_\lambda(\wp)(\Lambda, t) = \frac{\int_\Lambda [\wp(\Lambda, t)]^\lambda d\Lambda - 1}{1 - \lambda}, \quad \lambda \neq 1. \quad (12)$$

Fractional entropy introduces in natural way supplementary information as regards the implication of individual processes and to regulator modification. If λ has a large positive rate this measure is additional slight to records that often arise; nevertheless for large negative λ it is slighter to the processes which arise seldom. Obviously dimension methods are attractive considering a construction in the entropy calculations. The hypothetical work was surpassing to successfully differentiate dynamical systems expecting finite, noisy data, or to confirm a deterministic background. Accordingly, in these approaches and actions, the collection of data, which is naturally significant to

appreciate convergence, is impossibly large. By employing the fractional entropy on $\wp(\Lambda, t)$, we have the fractional entropy system [3]:

$$D_t^\nu T_\lambda(\wp)(\Lambda, t) = \frac{\partial}{\partial \Lambda} [\mu(\Lambda) T_\lambda(\wp)(\Lambda, t)] + \frac{1}{2} \frac{\partial}{\partial \Lambda} [\sigma^2(\Lambda) T_\lambda(\wp)(\Lambda, t)]. \quad (13)$$

D. Fractional Entropy in Fractional Space

The authors of the study [4] propose to consider the Boltzmann-Gibbs-Shannon entropy in the form:

$$S_{BG} = \langle \ln(1/p_i) \rangle \equiv k \sum_i p_i (-\ln p_i), \quad (14)$$

taking into account that p_i is a probability and k is a constant, one can perform transformations and in the new form represent expression (14) using the natural logarithm:

$$S = \langle \ln(1/p_i) \rangle_q \equiv \sum_i p_i (-\ln p_i), \quad (15)$$

Ubrico in [5] made a substitution of the known derivative for the fractional Weyl and received a new entropy record. In this case, the equation of fractional entropy assumed the form [4]:

$$S = \sum_i p_i (-\ln p_i) \equiv \langle (\ln(1/p_i))^q \rangle. \quad (16)$$

Here it is important to take into account that the expression obtained will be considered in a space of fractional order $(\hat{X}, A^\alpha, \Sigma)$ [6].

The relation of the fractal dimension with the fractional entropy of Tsallis illustrates the equation [7]:

$$D_k = \lim_{\varepsilon \rightarrow 0} \frac{K}{\ln \frac{1}{\varepsilon}} = \lim_{\varepsilon \rightarrow 0} \frac{\frac{1}{2k} \left(\sum_{i=1}^N p_i^{1-k} - \sum_{i=1}^N p_i^{1+k} \right)}{\ln \frac{1}{\varepsilon}}. \quad (17)$$

III. ENTROPY AND POINCARÉ RECURRENCE

A particularly interesting and deep connection is the one established between metric entropy and Poincaré recurrence. Given a measurable dynamical system f , it follows by pioneering work of Poincaré that the set of recurrent points has full probability. This means that the iterates of almost every point (with respect to an arbitrary invariant probability measure μ) will return arbitrarily close to itself. In particular, for any positive measure set A the function:

$$R_A(x) = \inf \{k \geq 1 : f^k(x) \in A\} \quad (18)$$

is finite almost everywhere in A / given a decreasing sequence of partitions U_n it is natural to look for a limiting behavior of the return times R_{U_n} in finer scales. Such a limiting behavior turned out to exist for ergodic stationary

processes and it coincides with the metric entropy of the system. More precisely, Ornstein and Weiss [3] proved that the entropy $h_\mu(f, Q)$ of an ergodic measure μ with respect to a partition Q is given by the (almost everywhere) well defined limit [3]:

$$h_\mu(f, Q) = \lim_{n \rightarrow \infty} \frac{1}{n} \log R_n(x, Q), \quad (19)$$

where $R_n(x, Q) = \inf \{k \geq 1 : f^k(x) \in Q^{(n)}(x)\}$ is the n th return time (with respect to the partition Q), $Q^{(n)} = \bigcup_{j=0}^{n-1} f^{-j}Q$ is the dynamically generated partition, and $Q^{(n)}(x)$ denotes the element of $Q^{(n)}$ that contains the point x .

Analysis of the gamma of parameters characterizing the interaction with an open system showed that it became possible to form a mathematical model of the type:

$$\tau_T^{\text{def}} = S_q(GM, d_f, \tau, \varepsilon, T_{st}, q - \exp), \quad (20)$$

where τ_T^α - thermodynamical spectrum of the dimensions of the Poincaré's return time in a open space; S_q - entropy of Tsallis; d_f - fractal dimensions in the function of entropy of Tsallis; τ - mean return time for Poincaré; ε - stability of Tsallis entropy; T_{st} - stability of thermodynamics; q - exp-exponential of Tsallis.

Thus, the resulting expression characterizes the thermodynamically category that is exchanged with a complex system in the framework of the transition process.

IV. ALGORITHM

A. Step 1

Let it place hyperjerk system [8].

A hyperjerk system is a dynamical system governed by an n -th order ordinary differential equation with $n > 3$ describing the time evolution of a single scalar variable.

$$\frac{d^4 x}{dt^4} + a_0 \frac{d^3 x}{dt^3} + a_1 \frac{d^2 x}{dt^2} + a_2 \frac{dx}{dt} = g\left(x, \frac{dx}{dt}, \frac{d^2 x}{dt^2}\right) \quad (21)$$

where $g\left(x, \frac{dx}{dt}, \frac{d^2 x}{dt^2}\right)$ - is nonlinear function.

B. Step 2

We consider the chaotic Q_i system [9]:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 x_4, \\ \dot{x}_2 = a(x_1 + x_2) - x_1 x_3 x_4, \\ \dot{x}_3 = -c x_3 + x_1 x_2 x_4, \\ \dot{x}_4 = -d x_4 + x_1 x_2 x_3, \end{cases} \quad (22)$$

where x_1, x_2, x_3 and x_4 are the state variables of the system and a, b, c and d are all positive real constant parameters.

A. Step 3

The multiplicative component of systems (21-22) under the influence of noise is shown in Fig.1.

Fig. 1. The results of the multiplicative mutual influence of the systems - a, the distance matrix - b and the calculation of its fractional dimension, $D=1.4053$.

The graph of the computed entropy of Tsallis is shown in Fig.2.

Fig. 2. The graph of Tsallis entropy – a, and its distance matrix – b.

The calculation of the values of the highest Lyapunov exponent is shown in Fig.3.

Fig. 3. The graph of the calculated values of the highest Lyapunov exponent (characterizes a chaotic process).

As for the relationship between the fractal dimension and the fractional entropy of Tsallis, then (23) [7]:

$$D_k = \lim_{\varepsilon \rightarrow 0} \frac{K}{\ln \frac{1}{\varepsilon}} = \lim_{\varepsilon \rightarrow 0} \frac{\frac{1}{2k} \left(\sum_{i=1}^N \frac{1}{N^{1-k}} - \sum_{i=1}^N \frac{1}{N^{1+k}} \right)}{\ln \frac{1}{\varepsilon}} \quad (23)$$

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