Re-reflection Effect on Shock Waves in Two Phase Flows through a Tube of Variable Cross Section

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***Abstract*—In present paper Re-reflection effect on shock waves in two phase flows through a tube of variable cross section is considered when particle volume fraction appears as an additional variable. It is concluded that re-reflected effects reduces the cross sectional area. For two-phase flows when equilibrium is eventually established, presence of particle volume fraction, further reduces the cross sectional area. One dimensional area relation for a non-uniform, steady flow ahead of a shock is obtained and concluded that all the results are valid for the case when direction of the shock motion and the gas flow ahead of the shock is same. (In preparation of graphs Mathematica 7 is used).**

***Index Terms*—Cross Sectional Area; Re-reflection; Shock Waves; Two Phase.**

I. INTRODUCTION

Flow of water, blood, gas and other fluids through a tube or channel of variable cross-section is important for daily life, medicine, engineering, underground explosion, floods and other branches of science and social sciences. If a shock moves along a channel or a tube with a small area change, the shock itself and the flow behind it are perturbed. When re-reflected disturbances generated by non-uniformity behind the shock are neglected, flow is called a freely propagating shock, as the shock wave is not affected by the re-reflected disturbances. Chisnell [1] and Whitham [2] have considered such type of problem separately using different methods and have obtained the relation between area of tube or channel and Mach Number.

Many researchers have studied the problem of re-reflected disturbances in the flow behind the moving shock. Rosciszewski [3] has formulated the error involved in using CCW approximation and obtained correction terms. Yousaf [4] has presented an exact formulation of the strength of the disturbances over taking the shock. Milton [5] has obtained a useful, simple relation between Mach number and area of the tube or channel.

The study of wave propagation in a mixture of gas and dust particles has received great attention during the last several decades. There are many engineering applications for flow of a medium that consists of a suspension of powdered material or liquid droplets in a gas. Dusty gas flows have importance in engineering problems such as flow in rockets, nuclear- reactors, fuel sprays, air pollution, etc. With the advancement of space technology, the dynamics of fluid particle system has found applications in extra-terrestrial field such as lunar-ash-flow and predictably in the studies of other planets. The dynamics of dusty gas is modified from conventional gas dynamics by characterizing the temperature and velocity of the gas and particle separately. A single particle that is not in equilibrium with the gas flow simply represents a poor ‘tracer’ but if there are enough particles to form a significant fraction of the mass of the mixture, their interaction with the gas affects the gas flow, rather complicated flows can therefore develop as a result of the relaxation processes. As in the case of pure gas flows, the rate at which deviations from equilibrium tend to be eliminated may be fast or slow compared with the rate at which flow changes take place. It is therefore possible to consider ‘frozen’ flow in which no relaxation processes take place, equilibrium flows for which relaxation is assumed to be infinitely fast, and intermediate non equilibrium flows.

Along with advances in various flow fields mentioned above, some earnest efforts have been made in understanding the behavior of dusty gas, starting with many simplifying assumptions and modifications. The paper by Marble [6] was an attempt in applying the modern techniques of fluid mechanics to the analysis of dusty flows. He has introduced many important concepts and parameters which can be served as strong via media in the development of the fundamental equations of the mixture of gas and solid particles. Marble [7] provide an extensive study of the flows of the dusty gas with example of shock formation. Rudinger [8] has presented the thermodynamic properties of shock waves, steady nozzle flow and general non steady one-dimensional flow of the gas particle mixture with various examples depicting the importance of velocity and temperature relaxations. Jena and Sharma [9] have studied the self-similar shocks in dusty gases. Following Whitham [2], Pandey and Verma [10] have discussed the formation of shock down a non-uniform tube in two phase flows.

The mathematical analysis of such two phase flow is considerably more difficult than that of pure gas flows and one of the usual simplifying assumptions is that the volume occupied by the particles can be neglected. In many important cases, the particle represents less than one half of the mass of gas particle mixture and the density of the particle material is more than thousand times larger than the gas density. Under such conditions the particle volume fraction is of order of 10^{-4} and assumption of a negligible particle volume is then well satisfied. One more important consequence of this assumption is that equilibrium flow of the mixture of particles with a perfect gas can be analyzed like flow of perfect gas that has density and specific heats of mixture. Carrier [11] was first to study the motion of shock wave in dusty gases. Various aspects of two-phase flows were studied by Soo [12], Kribe [13], Rudinger [14], Marble [15], Bailey [16], Kliegel [17], Gilbert [18], Kliegel [19].

At high gas densities (high pressure) or at high particle

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mass fractions, the particle volume fraction may become sufficiently large, so that it may be included into flow analysis without introducing significant error. Since the particles may be considered as incompressible in comparison with the gas, the particle volume fraction enters into the basic flow equations as an additional variable. The interesting properties of such two phase flows is that even equilibrium flows cannot be treated as perfect gas flows. There are many engineering problems in which dilute phase of gas particles is a good approximation of actual conditions. In such cases due to the existence of solid particles in the gas, properties of mixture differ significantly from those of gas alone. Such types of studies have numerous applications in underground explosion [20], [21].

In present paper Re-reflection effect on shock waves in two phase flows through a tube of variable cross section is considered when particle volume fraction appeared as an additional variable. Firstly, Re-reflection effects on shock wave in a tube of variable cross section is obtained and secondly, one dimensional area relation for a non-uniform, steady flow ahead of a shock is obtained and concluded that all the results are valid for the case when direction of the shock motion and the gas flow ahead of the shock is same.

II. Basic Equations

We consider one dimensional formulation for flow in a tube of a given cross-sectional area A, where $A(x)=A_0$=constant, in x < 0, and the shock is initially moving in this section with a constant Mach number. We consider that the shock is to be produced by a piston moving with appropriate constant speed far back in the uniform section. The piston is still providing the thrust to keep the shock moving, but there are no changes due to this and the changes are entirely due to the cross-sectional area. Though the flow is not strictly one dimensional but if the cross section does not vary too rapidly, the equations obtained by averaging across the tube will provide a good approximation of one dimensional flow. Thus equations governing the motion are given by Rudinger [8],

$$A(\sigma+\sigma_p),_t+(\sigma uA),_x+(\sigma_p u_p A),_x=0\ , \quad (1)$$

$$\sigma(u,_t+uu,_x)+\sigma_p(u_{p,t}+uu_{p,x})+p_{,x}=0\ , \quad (2)$$

$$A\left[\sigma\left(\frac{u^2}{2}+C_vT\right)+\sigma_p\left(\frac{u_p^{\,2}}{2}+CT_p\right)\right]_{,t}+\left[\begin{array}{l}\sigma uA\left(\frac{u^2}{2}+C_pT\right)+\\ \sigma_p u_p A\left(\frac{u_p^{\,2}}{2}+CT_p+\varepsilon\frac{p}{\sigma_p}\right)\end{array}\right]_{,x}=0\ , \quad (3)$$

$$A\varepsilon,_t+(A\varepsilon u),_x=0\ , \quad (4)$$

where σ,u,T,p,C_p,C_v are gas concentration, velocity, temperature, pressure , specific heat at constant pressure and at constant volume $\sigma_p,u_p,T_p,C,\varepsilon$ are particle concentration, velocity, temperature, specific heat and particle volume fraction and A is duct area.. A comma followed by an index implies partial differentiation with respect to that index.

Equation of state in present case can be written as,

$$p=\frac{\rho_m R_m T}{1-\varepsilon},$$

where, $\rho_m=(1-\varepsilon)(1+\eta)\rho$ is density of mixture and $R_m=(1-\phi)R$ is effective gas constant for mixture , η,ϕ being mass flow ratio and mass flow rate respectively.

Using above equation of state, (1) to (4) can be re-written in the following form,

$$(1-\varepsilon)\rho,_t+u(1-\varepsilon)\rho,_x+\rho u,_x+\frac{\rho u(1-\varepsilon)}{A}A,_x=0\ , \quad (5)$$

$$u,_t+uu,_x+\frac{1}{\rho(1+\eta)(1-\varepsilon)}p,_x=0\ , \quad (6)$$

$$p,_t+up,_x-a_e^{\,2}\{\rho,_t+u\rho,_x\}=0\ , \quad (7)$$

$$\varepsilon,_t+u\varepsilon,_x+\varepsilon u,_x+\frac{\varepsilon u}{A}A,_x=0\ , \quad (8)$$

where $a_e^{\,2}=\frac{p\gamma_m}{\rho_m(1-\varepsilon)}$ and $\gamma_m=\frac{\gamma(1+\eta\xi)}{(1+\gamma\eta\xi)}$ with $\xi=\frac{C}{C_p}$ are equilibrium sound speed and ratio of specific heats for mixture.

III. Conservation Laws and Shock Relations

Conservation of mass, momentum and energy for two phase flows when equilibrium is established eventually and particle volume fraction is taken as an additional variable are given by Rudinger [22],

$$(1-\varepsilon)\rho uA=(1-\varepsilon_0)\rho_0 u_0 A=m\ , \quad (9)$$

$$\varepsilon\rho_p u_p A=\varepsilon_0\rho_p u_0 A=\eta m\ , \quad (10)$$

$$mu+\eta mu_p+Ap=m(1+\eta)u_0+Ap_0\ , \quad (11)$$

$$\frac{u^2}{2}+C_pT+\eta\left\{\frac{u_p^2}{2}+CT_p+\frac{p}{\rho_p}\right\}=(1+\eta)\frac{u_0^2}{2}+(C_p+\eta C)T_0+\eta\frac{p_0}{\rho_p}\ , \quad (12)$$

At equilibrium, we can write $u = u_p = u_e$ and $T = T_p = T_e$ thus (9) to (12) reduces to following set of equations,

$$(1-\varepsilon_e)\rho_e u_e A = (1-\varepsilon_0)\rho_0 u_0 A = m \ , \tag{13}$$

$$\varepsilon_e u_e A = \varepsilon_0 u_0 A = \eta m \ , \tag{14}$$

$$m(1+\eta)u_e + Ap_e = m(1+\eta)u_0 + Ap_0 \ , \tag{15}$$

$$(1+\eta)\frac{u_e^{\ 2}}{2} + (C_p + \eta C)T_e + \eta\frac{p_e}{\rho_e} = (1+\eta)\frac{u_0^{\ 2}}{2} + (C_p + \eta C)T_0 + \eta\frac{p_0}{\rho_0} \ . \tag{16}$$

Shock conditions in present case are given by following set of equations

$$\frac{u_e}{u_0} = \frac{(\gamma_m - 1)M_e^{\ 2} + 2 + 2\varepsilon_0(M_e^{\ 2} - 1)}{(\gamma_m + 1)M_e^{\ 2}} \ , \tag{17}$$

$$\frac{\varepsilon_e}{\varepsilon_0} = \frac{u_o}{u_e} \ , \tag{18}$$

$$\frac{\rho_e}{\rho_0} = \left\{\frac{(1-\varepsilon_0)}{(1-\varepsilon_e)}\right\}\frac{u_0}{u_e} \ , \tag{19}$$

$$\frac{p_e}{p_0} = 1 + \frac{\gamma u_0(u_0 - u_e)(1+\eta)(1-\varepsilon_o)}{a_0^{\ 2}} = 1 + \frac{2\gamma_m(M_e^{\ 2} - 1)}{(\gamma_m + 1)} \ , \tag{20}$$

where subscript e denotes the quantities when equilibrium is established.

For present case moving shock relations are given by following equations

$$p_2 = p_1\left[\frac{2\gamma_m M^2}{\gamma_m + 1} - \frac{\gamma_m - 1}{\gamma_m + 1}\right] , \tag{21}$$

$$\rho_2 = \frac{\rho_1(\gamma_m + 1)M^2}{(\gamma_m - 1)M^2 + 2 + 2\varepsilon_1(M^2 - 1)} \ , \tag{22}$$

$$u_2 = 2a_1\frac{(1-\varepsilon_1)}{\gamma_m + 1}\left(M - \frac{1}{M}\right) , \tag{23}$$

$$a_2 = \frac{a_1}{M(\gamma_m + 1)}\sqrt{\frac{\left\{2\gamma_m M^2 - (\gamma_m - 1)\right\}}{\left\{(\gamma_m - 1)M^2 + 2 + 2\varepsilon_1(M^2 - 1)\right\}}} , \tag{24}$$

$$\frac{p_2 - p_3}{p_1} = \frac{2\gamma_m}{\gamma_m + 1}\left(M^2 - M_0^2\right) , \tag{25}$$

where subscript e is dropped out for convenience.

IV. Formation of Problem

If a shock wave moves along a channel (or tube) with a small area change, the shock itself and the flow behind it are perturbed. Chisnell [1] and Whitham [2] have considered such a type of problem separately using different methods. But in their methods re-reflected disturbances generated by non-uniformity behind the shock is not considered. Milton [5] has modified the method of Whitham [2] by taking into account the interaction term. He has described the flow pattern of the motion of a shock wave through a slowly varying cross sectional area of tube as follows:

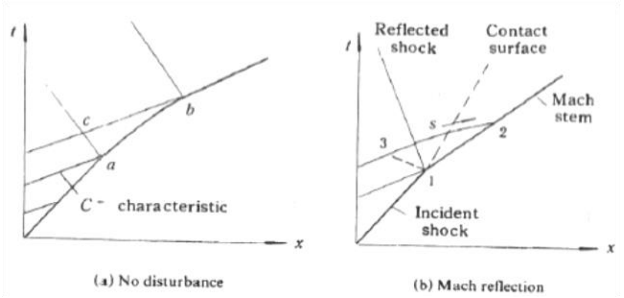

Fig. 1. Milton's Description

The incident shock is disturbed by the flow behind it and a reflected shock, contact surface and Mach stem are formed (Fig 1). In (Fig. 1) line 32 follows characteristic C+, but due to the interaction of the reflected shock and the contact surface values along it differ from those given by Whitham [2].

Calculation of Correction term

In Fig. 1 line 31 is not a characteristic but connects and arbitrary point 3 in the region just prior to the formation of the reflected shock to point 1. Thus according to Milton [5], the correction term on the shock front can be written as:

$$(\varsigma)_1^2 = (\varsigma)_3^2 + (\varsigma)_1^3 \ , \tag{26}$$

where,

$$d\varsigma = du + \frac{1}{\rho a}dp + \frac{ua}{u+a}d(\log A) \ , \tag{27}$$

and u, a, ρ ,p are velocity, sound velocity, density, pressure of two phase flows and A is variable cross sectional area of tube or channel .On line 31,the incident shock is undisturbed and du ,dp ,d(logA) are zero, thus (26) reduces to,

$$(\varsigma)_1^2 = (\varsigma)_3^2 \ . \tag{28}$$

In order to find the values, for the disturbance terms on the line 32, the function (dς)is assumed as an approximation, to be continuous, not zero. Integrating (28) between end

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points (points 2 and 3) and differentiating with respect to s in the characteristic direction we have,

$$\frac{1}{ds}\int_3^2 d\varsigma = \frac{d\varsigma_2}{ds} - \frac{d\varsigma_3}{ds} + \chi + \tau \quad , \tag{29}$$

where,

$$\chi = \int_3^2 \frac{\partial}{\partial s}\left(\frac{1}{\rho a}\right)dp \quad \text{and} \quad \tau = \int_3^2 \frac{\partial}{\partial s}\left(\frac{ua}{u+a}\right)d(\log A) \quad . \tag{30}$$

At either end of 2 or 3, the line represents a true characteristic and hence characteristic identity holds, thus $\frac{d\varsigma_2}{ds}$ and $\frac{d\varsigma_3}{ds}$ are equal to zero. In the undisturbed case $d\varsigma = 0$ or $\frac{d\varsigma}{ds} = 0$,holds for whole line23, which imply that χ and τ also vanish. But in the disturbed case, χ and τ are not equal to zero, hence correction term or interaction term is generated and can be given as,

$$(d\varsigma)_1^2 = (\chi + \tau)ds \tag{31}$$

or,

$$du_2 + \frac{dp_2}{\rho_2 a_2} + \frac{u_2 a_2}{u_2 + a_2} d(\log A) = (\chi + \tau)ds \quad , \tag{32}$$

where, subscript 2 represents the flow immediately behind the moving shock and can be substituted from Rankine-Hugoniot conditions, given by (21) to (24).

From (30) we have,

$$d\chi = \frac{\partial}{\partial s}\left(\frac{1}{\rho a}\right)dp \quad \text{and}$$
$$d\tau = \frac{\partial}{\partial s}\left(\frac{ua}{u+a}\right)d(\log A) \tag{33}$$

or,

$$-\frac{d\chi}{dp} = -\frac{\partial}{\partial s}\left(\frac{1}{\rho a}\right) = \left(\frac{1}{\rho a^2}\right)\frac{\partial a}{\partial s} + \left(\frac{1}{\rho^2 a}\right)\frac{\partial \rho}{\partial s} \tag{34}$$

and

$$-\frac{d\tau}{d\log A} = -\frac{\partial}{\partial s}\left(\frac{ua}{u+a}\right) = \frac{u^2\left(\frac{\partial a}{\partial s}\right) + a^2\left(\frac{\partial u}{\partial s}\right)}{(u+a)^2} \quad . \tag{35}$$

The quantities given by (34) and (35) can be evaluated in the region upstream of the reflected shock and at the main shock front. As at point 3,

$\frac{\partial a_3}{\partial s}$, $\frac{\partial \rho_3}{\partial s}$ and $\frac{\partial u_3}{\partial s}$ are all zero. Hence,

$$\left(\frac{d\chi}{dp}\right)_3 = \left(\frac{d\tau}{d\log A}\right)_3 = 0 \quad .$$

At point 2, it is assumed that variation in the s direction can be approximated by variations in the shock front direction, hence Rankine-Hugoniot conditions on the shock front can be used to evaluate (33) or (34) and (35).

Thus under strong shock conditions we have,

$$\chi = -\frac{1}{2}\left(\frac{(\gamma_m - 1) + 2\varepsilon_1}{2\gamma_m}\right)^{1/2} \frac{1}{\rho_1 a_1 M^2}(p_2 - p_3)\frac{dM}{ds} \tag{36}$$

and

$$\tau = \frac{2a_1}{(\gamma_m + 1)} \left(\frac{\{2\gamma_m(\gamma_m - 1 + 2\varepsilon_1)\}^{1/2} + \gamma_m(\gamma_m - 1 + 2\varepsilon_1)}{[\{2\gamma_m(\gamma_m - 1 + 2\varepsilon_1)\}^{1/2} + 2]^2}\right) \left\{\log\left(\frac{A_2}{A_3}\right)\frac{dM}{ds}\right\} \quad , \tag{37}$$

where subscript 1, represents the flow condition ahead of the main shock, the subscript 2 represents the flow condition behind the disturbed shock (Mach Stem) whose Mach number is M and subscript 3 represents the flow condition ahead of the reflected shock, which just corresponds to the flow condition behind the incident shock, whose Mach number is M_0.

Equation (32) is valid along the moving shock, and moving shock relations are given by (21) to (24).

Considering the condition used by Miura and Glass [23], we have following relations,

$$du_2 = \frac{2a_1(1-\varepsilon_1)}{\gamma_m + 1}\left(1 + \frac{1}{M^2}\right)dM \quad , \tag{39}$$

$$dp_2 = \frac{4p_1\gamma_m M dM}{\gamma_m + 1} \quad , \tag{40}$$

$$\rho_2 a_2 = \frac{\rho_1 a_1 M}{\mu} \quad , \tag{41}$$

where

$$\mu = \sqrt{\frac{(\gamma_m - 1)M^2 + 2 + 2\varepsilon_0(M^2 - 1)}{2\gamma_m M^2 - (\gamma_m - 1)}} \quad .$$

Substituting values from (21)-(24) and Relations (39)–(41), in (32) it reduces into following equation,

$$\frac{dA}{A}=-\left(\frac{M}{(M^2-1)g(M)}+\frac{\delta}{M}\right)dM \quad ,$$
where
$$\delta=.\frac{1}{2\gamma_m}\left((1+\varepsilon_1)+\sqrt{\frac{\gamma_m\{(\gamma_m-1)+2\varepsilon_1\}}{2}}\right)\left((1-\frac{M_0^2}{M^2})\frac{1}{(1-\varepsilon_1)}+\frac{1}{2}\log\frac{A_0}{A}\right) \tag{42}$$

and

$$g(M)=\frac{1}{(1-\varepsilon_1)}\left\{2\mu+(1-\varepsilon_1)\left(1+\frac{1}{M^2}\right)\right\}^{-1}\left\{1+\left(\frac{2(1-\varepsilon_1)}{\gamma+1-2\varepsilon_1}\right)\left(\frac{1-\mu^2}{\mu}\right)\right\}^{-1}$$

and (42) represents the variation of area of tube when interaction term is included.

V. Area Relation for a Non-uniform, Steady Flow Ahead of a Shock

In this section for originally steady two phase flows, one dimensional area relation for a non-uniform, steady flow ahead of a shock is considered when moving shock is propagating through a varying cross sectional area of tube or channel. The compatible relation along the positive characteristics behind moving shock is given by,

$$dp_2+\rho_2 a_2 du_2+\frac{\rho_2 a_2^2 u_2}{u_2+a_2}\frac{dA}{A}=0 \tag{43}$$

or,

$$\frac{dp_2}{\gamma_m p_2}+\theta_2\frac{du_2}{u_2}+\frac{\theta_2}{\theta_2+1}\frac{dA}{A}=0 \quad . \tag{44}$$

where, subscript 2 represents the state behind the moving shock , θ is the flow Mach number behind the moving shock and other variables have the same meaning as given in section two and four.

Moving shock relation in present case is given as follows:

$$\frac{p_2-p_1}{p_1}=\frac{2\gamma_m}{\gamma_m+1}(M^2-1) \quad , \tag{45}$$

$$\frac{u_2-u_1}{u_1}=\frac{2}{(\gamma_m+1)}\frac{1}{\theta_1}(M-\frac{1}{M}) \quad , \tag{46}$$

$$\theta_2=\frac{(\gamma_m+1)\theta_1 M+2(M^2-1)}{[2\gamma_m M^2-(\gamma_m-1)]^{1/2}[(\gamma_m-1)M^2+2]^{1/2}} \quad . \tag{47}$$

The relations of homoentropic, steady flow ahead of the shock are expressed as,

$$\frac{p_1}{p_T}=\left(1+\frac{(\gamma_m-1)}{2}\theta_1^2\right)^{\frac{-\gamma_m}{(\gamma_m-1)}} \quad , \tag{48}$$

$$\frac{u_1}{a_T}=\frac{u_1}{a_1}\frac{a_1}{a_T}=\theta_1\left(1+\frac{(\gamma_m-1)}{2}\theta_1^2\right)^{-1/2} \quad , \tag{49}$$

$$\frac{dA}{A}=\frac{(\theta_1^2-1)}{1+\frac{(\gamma_m-1)\theta_1^2}{2}}\frac{d\theta_1}{\theta_1} \quad , \tag{50}$$

where p_T and a_T are stagnation parameters , θ_1 is the flow Mach number in the region ahead of the shock.

With help of (45) and (48) we have,

$$p_2=p_T T(\theta_1)^{-\frac{\gamma_m}{\gamma_m-1}}\left(\frac{2\gamma_m}{\gamma_m+1}M^2-\frac{\gamma_m-1}{\gamma_m+1}\right) \quad , \tag{51}$$

where

$$T(\theta_1)=1+\frac{(\gamma_m-1)}{2}\theta_1^2.$$

Differentiating above equation we have,

$$dp_2=-\gamma_m p_T\left(\frac{2\gamma_m}{(\gamma_m+1)}M^2-\frac{(\gamma_m-1)}{(\gamma_m+1)}\right)T(\theta_1)^{-\frac{2\gamma_m-1}{\gamma_m-1}}\theta_1 d\theta_1+\frac{4\gamma_m}{(\gamma_m+1)}p_T T(\theta_1)^{-\frac{\gamma_m}{\gamma_m-1}}MdM \quad . \tag{52}$$

Thus from (51) and (52),we have,

$$\frac{dp_2}{\gamma_m p_2}=\frac{4MdM}{[2\gamma_m M^2-(\gamma_m-1)]}-\frac{\theta_1 d\theta_1}{T(\theta_1)} \quad . \tag{53}$$

Similarly, from (46) and (49), we have

$$u_2=a_T T(\theta_1)^{-1/2}\left(\frac{2(M^2-1)}{(\gamma_m+1)M}+\theta_1\right) \quad . \tag{54}$$

Differentiating (54), we have

$$du_2=a_T T(\theta_1)^{-1/2}\left(1-\frac{(\gamma_m-1)}{(\gamma_m+1)}\theta_1\left\{\frac{M^2-1}{M}\right\}T(\theta_1)^{-1}-\frac{(\gamma_m-1)}{2}\theta_1^2 T(\theta_1)^{-1}\right)d\theta_1+\frac{2a_T T(\theta_1)^{-1/2}}{(\gamma_m+1)}\left(1+\frac{1}{M^2}\right)dM \quad , \tag{55}$$

$$\theta_2 \frac{du_2}{u_2} = \frac{\theta_2}{S}\begin{pmatrix} 1-\frac{(\gamma_m-1)}{(\gamma_m+1)}\theta_1\left\{\frac{M^2-1}{M}\right\}T(\theta_1)^{-1} \\ -\frac{(\gamma_m-1)}{2}\theta_1^2 T(\theta_1)^{-1} \end{pmatrix} d\theta_1 + \frac{\theta_2}{S}\frac{2}{(\gamma_m+1)}\left(1+\frac{1}{M^2}\right)dM \qquad (56)$$

where,

$$S = S(M,\theta_1) = \frac{2(M^2-1)}{(\gamma_m+1)M} + \theta_1 \quad .$$

Substituting values in (44), with help of (47),(50),(53) and (56), we have following relations for one dimensional shock moving through a tube with non-uniform two phase flows as,

$$dM = G d\theta_1 \quad \text{and}$$

$$\frac{dA}{A} = \frac{(\theta_1^2-1)}{1+\frac{(\gamma_m-1)\theta_1^2}{2}}\frac{d\theta_1}{\theta_1} \quad , \qquad (57)$$

where,

$$G = \left[\frac{\frac{\theta_1}{T}+\left(\frac{\gamma_m-1}{2}\right)\theta_1\frac{\theta_2}{T}-\frac{\theta_2}{S}-\left(\frac{\theta_1^2-1}{\theta_1 T}\right)\left(\frac{\theta_2}{\theta_2+1}\right)}{\frac{2\theta_2}{(\gamma_m+1)S}\left(1+\frac{1}{M^2}\right)+\frac{4M}{\{2\gamma_m M^2-(\gamma_m-1)\}}}\right],$$

when the direction of the shock motion is same as that of the gas flow ahead of the shock, G>0.

Above relation shows that for supersonic flow ahead of the shock in case of varying cross-sectional area of tube, dA > 0, dθ_1 > 0, dM > 0 and dA < 0, dθ_1 < 0, dM < 0.

For subsonic flow dA > 0, dθ_1 < 0, dM < 0 and dA <0 , dθ_1 > 0, dM > 0.

Above conclusion is valid only under the condition that the shock wave and the flow ahead of it have same direction.

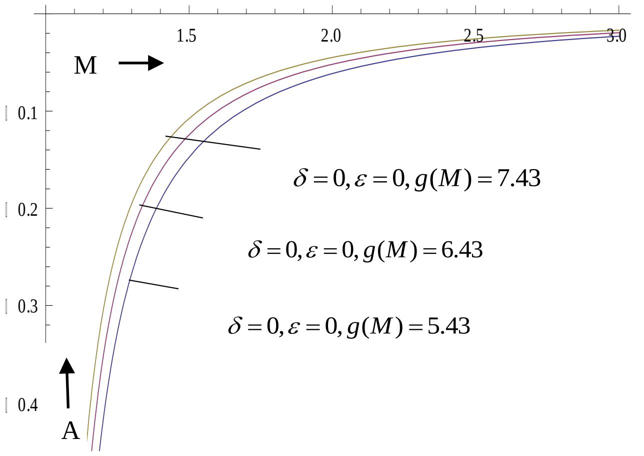

Fig 1. A graph between the Mach number *M* and Area *A* for different values of *g(M)*

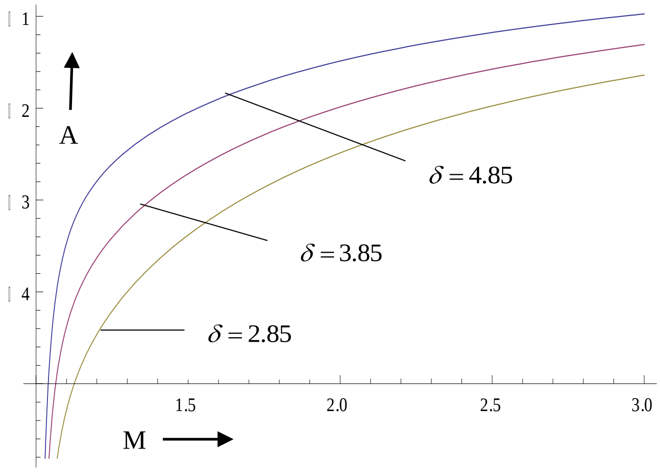

Fig. 2. A graph between the Mach number *M* and Area *A* for different values of δ or fix particle volume fraction ε=0 and *g(M)*=5.43

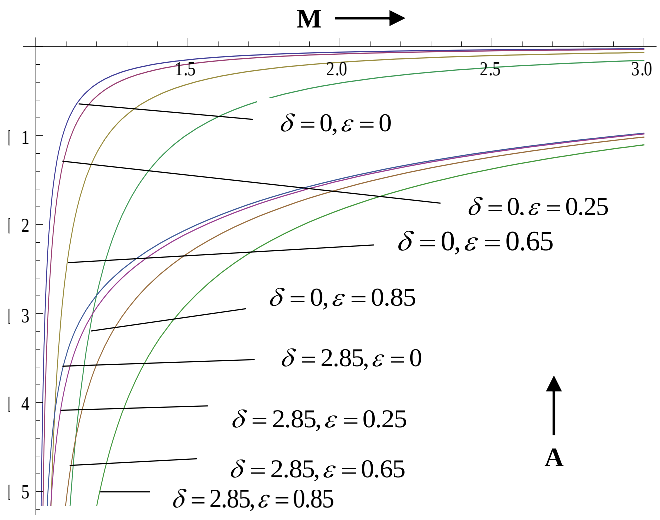

Fig 3. A graph between the Mach number M and Area A for different values of particle volume fraction ε and δ for fix g(M)=5.43

VI. Results and Discussion

In this article a relationship between cross sectional area (A) and Mach number (M) is obtained when re-reflection effect is taken into account while a shock moves along a channel or tube with a small area change. Various cases with and without re-reflected effect have been considered and it

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is concluded that re-reflection effect reduces the cross sectional area. It is concluded that if we increase the value of particle volume fraction, the cross- sectional area gets further reduced (Fig. 3). If we take δ =0, the problem reduces to that considered by Pandey and Verma[10]. For $\varepsilon = 0$, it reduces into problem dealt by Milton[5] (Fig. 2). If both δ =0 , $\varepsilon = 0$, problem reduces to that considered by Whitham[2] and Chisnell[1] (Fig. 1).

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REFERENCES

[1] R.F. Chisnell "The normal motion of a shock wave through a non-uniform one dimensional medium," *Proc. Roy.Soc.Ser.A,* vol.232, pp.350-370, (1955).
[2] G.B. Whitham, "Linear and non-linear waves," Wiley Interscience, New York, (1973).
[3] J. Rosciszewski, "Calculation of the motion of non-uniform shock waves," Joural of Fluid Mechanics, vol.8, pp. 337-367, (1960).
[4] M. Yousaf, "The effect of overtaking disturbances on the motion of converging shock waves," *Journal of Fluid Mechanics*, vol.66, pp. 577-59, (1974).
[5] B.E. Milton, "Mach reflection using ray shock theory," *AIAA Journal*, vol.13, pp.1531-1533, (1975).
[6] F.E. Marbel, "Dynamics of dusty gases," *Ann. Rev. Fluid Mech,* vol.2, pp. 397-446, (1970).
[7] F.E. Marbel, "Nozzle contours for minimum particle- lag loss," *AIAAJ*, vol., pp.2793-2801, (1963).
[8] G. Rudinger, "Relaxation in gas particle flow; Non Equilibrium Flows: Part I," Ed. P. P. Wegener, Marcel Dekker Press, pp.119-159, (1969).
[9] J.Jena and V.D. Sharma, "Self Similar Shocks in a Dusty Gas," *International J. of Non-linear Mechanics*, vol.34, pp. 313-327, (1999).
[10] K. Pandey and P. Verma, "Shock formation down a non-uniform tube in two Phase Flow," *Journal of Mathematics Research*, vol. 5 (No 3), pp. 17-25, (2013).
[11] G.F. Carrier "Shock waves in dusty gas," *J. of Fluid Mech*, vol.4, pp.376-382, (1958).
[12] S.L. Soo, "Gas dynamic processes involving suspended solids," A.I.Ch.E.J., vol.7(No.3), pp.384-391, (1961).
[13] A.R. Kriebel, "Analysis of normal shock waves in particle laden gas," *J. Basic Eng.*, vol.86, pp.655-665, (1964).
[14] G. Rudinger, "Some properties of shock relaxation in gas flows carrying small particles," *Physics of Fluid*, vol.7, pp.658-663, (1964).
[15] F.E. Marbel, "Nozzle contours for minimum particle- lag loss," *AIAAJ*, vol., pp.2793-2801, (1963).
[16] W. S. Bailey and R. A.Serra " Gas particle flow in an axis symmetric nozzle," *ARSJ*, vol.3, pp, 793-799, (1961).
[17] J.R. Kliegel, "Gas particle nozzle flow symp. Combust," *9th Academic New York*, pp.811-826, (1963).
[18] M. Gilbert, J. Allport and R. Dunlop, "Dynamics of two phase flow in rocket nozzles," *ARSJ*, vol.32(no.12), pp.1929-1930, (1962).
[19] J.R. Kliegel, "One dimensional flow of gas particle system," *IAS Paper No. 60-5, 28th Annual Meeting of I. Aero. Sci., New York*, (1960).
[20] F.K. Lamb, B.W. Collen, and J.D. Sullivan, "An approximate analytical model of shock waves from underground nuclear explosion," *J. Geophys. Res.*, vol. 97, pp.515-535, (1992) .
[21] K. Nagayama, "Shock wave interaction in solid material,"A.B. Sawaoka, (Ed.) "Shock waves in material science," *Springer –Berlin*, (1993).
[22] G. Rudinger, "Some effects of finite particle volume on the dynamics of gas particle mixture,"*AIAAJ*, vol.3, pp.1217-1222, (1965).
[23] H.Miura and I.I. glass, "Development of the flow induced by a piston moving impulsively," *Proc. Roy. Soc. London*, vol. 397, pp. 295-309, (1985).

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